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Rainfall Intensity-Duration-Frequency Models for Ibadan Using Optimization Technique

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Abstract: Hydraulic engineering structures always depend on type and velocity of flow water arising from rainfall activities in a given catchment. Twenty-five (25) year Ibadan daily rainfall data (amount & duration) was collected from Nigerian Meteorological Agency (NIMET) Abuja and subjected to frequency analysis for the development of intensity – duration – frequency models. Mean rainfall amounts with durations of 5, 10, 15, 20, 30, 45, 60, 90, 120, 180, 240, 300 and 420 minutes were extracted and subjected to frequency analysis using the Excel Optimization Solver wizard. Gumbel Extreme Value Type 1 and Log- Pearson Type 3 distributions were used to develop specified and non-specified IDF models for return periods of 2, 5, 10, 25, 50 and 100 years. These models have not been developed for Ibadan. The values for the coefficient of determination (R^2) and mean squared error (MSE) were used to test the fitness of the probability distribution functions. Gumbel Extreme Value Type – 1 has values of R^2 and MSE ranging from 0.989 to 0.998 & 3.54 to 102.30 while R^2 and MSE values for Log – Pearson type 3 distribution ranges from 0.978 to 0.989 & 6.09 to 213.10. The probability distribution models are recommended for the prediction of rainfall intensities of Ibadan metropolis for the Ministry of Works for adequate design purposes.

Keywords: Excel Optimization Solver, Gumbel Extreme Value Type I, Goodness of fit test, IDF models, Log Pearson Type 3 distributions.

I. Introduction

The adequate management of water resources in all river basins can be used as a tool to meet the United Nation's Sustainable Development Goals number one and two (Eradicate poverty and food security). The use of statistical analysis tools on rainfall amount and duration leads to the development of Rainfall Intensity Duration Frequency (IDF) relationships which can be used for effective analysis and design of flood control structures. [1] recorded that good planning and design activities for extreme events like flood, drought, high wind and rainstorm are better achieved when there is adequate knowledge of extreme event frequency. [2] Opined that projects in water resources depend on adequate knowledge of rainfall intensity – duration – frequency (IDF) modelling. Probability distribution functions are used for frequency analysis of rainfall amount and duration from rain gauge stations. The IDF formulae are the empirical equations representing a relationship among dependent variable (maximum rainfall intensity) and other parameters of interest which represents independent variables (for example rainfall duration and frequency). [3], stated that a number of these probability functions are found in hydrological applications in real life. Accurate estimation of intensity-duration-frequency relationship has received attention from researchers and scientists from all over the world because of its wide application [4]. The coverage in Nigeria has extended to South – East and South-South from North

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Central such as the IDF models of Port Harcourt [5 - 6] and that of Eket in Awka Ibom State [7]. The theory of IDF is upheld in these results for the return period ranging from 2 to 10 years. Adequate knowledge of IDF helps in achieving a climate smart agricultural practice. The management of natural resources (water and land) is crucial in eliminating challenges faced by small-scale farmers which include access to technical know-how, low market access and limited investment [8].

The development of these models is important for the adequate design of flood mitigating structures to avoid incidences of flooding in Ibadan as recently experienced in 2019 with attendant loss of lives and property worth several millions of Naira.

II. Materials and Methods

A. Study Area Description

The capital of Oyo State, Ibadan located in South – West Nigeria with a distance of about 128km inland northeast of Lagos and 530 km southwest of Abuja and covering an estimated metropolitan area of about 3,080 km^2. The elevation is 230m above the sea level and falls within latitude 7° 23´ 47”N and longitudes 3° 55´ 0”E. Ibadan lies completely within the tropical forest zone but close to the boundary between the forest and the derived savannah. The mean annual temperature and rainfall of Ibadan metropolis is 28^{0}C & 1,375mm and it's properly drained by rivers Ona, Ogbere, Ogunpa and Kudeti. The study area is graphically illustrated in Figure 1.

B. Data Collection

The Nigerian Meteorological Agency (NIMET) provided data covering 1986 to 2010 (25 years) which was sorted into durations ranging from five (5) to four hundred and twenty (420) minutes. The computation of rainfall intensities for the development of the various models was achieved by using the ranked data.

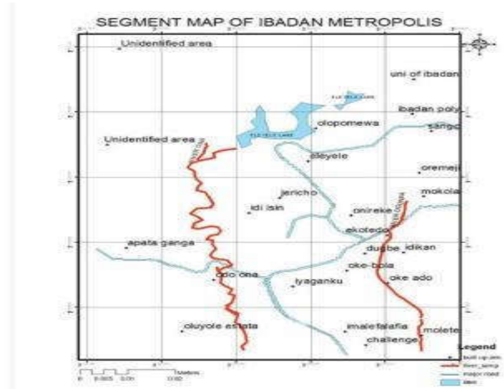

Figure 1: Map of Ibadan Metropolis and environ
Source [9]

C. Data Analysis

The maximum rainfall amount was selected for the following durations: 5, 10, 15, 20, 30, 45, 60, 90, 120, 180, 240, 300, and 420 durations (minutes).

The IDF relation expressed by [10 - 11] is mathematically expressed as shown in Equation (1):

$$I = f(T,d) \quad (1)$$

Note: intensity is I; return period, T and duration, d.

The rainfall amount is converted to intensity (mm/hr) by dividing the amount by the duration (minutes) then multiplying by 60 as a conversion factor. For instance, given rainfall amount of 36.6 mm for 10-minute duration yields an intensity of (36.6/10) x 60 = 219.6 mm/hr.

Table 1 shows all the intensities for various durations for Ibadan.

Frequency analysis tool was used for obtaining rainfall intensities magnitude for the station under consideration. The earlier study by [6] showed that Gumbel Extreme Value Type I and Log-Pearson Type III best fits rainfall models that has been developed; hence these two probability distribution functions were adopted for calculating the rainfall intensities for selected return periods

D. . Gumbel Extreme Value Type I (GEVT- 1) Distribution

GEVT – I Gumbel distribution is one commonly used probability distribution for obtaining the rainfall intensity values. The rainfall intensity values were obtained using Equation (2):

$$X_T=\bar{X}+K_TS \tag{2}$$

Where

X_T = rainfall intensity values (magnitude of the hydrologic event) I = rainfall intensity, C, m and a = model parameters, T_r = return period (year) and T_d = duration (hours)

Table 1: Ranked Observed Annual Rainfall Intensities (mm/hr) for different Durations (minutes)

	Rainfall intensity (mm/hr)												
Year	5	10	15	20	30	45	60	90	120	180	240	300	420
1	333.3	219.6	170.8	143.1	108.4	84.0	69.5	50.7	40.1	30.6	26.8	24.4	26.2
2	274.8	209.9	160.2	133.8	106.2	79.5	69.4	48.9	40.0	30.1	25.2	21.7	17.4
3	258.0	205.2	159.2	132.3	104.4	77.5	66.9	48.5	39.6	26.6	23.0	21.4	17.4
4	221.7	180.6	149.2	128.1	100.9	77.0	66.2	48.3	39.3	26.4	22.6	18.4	16.6
5	219.6	153.0	136.8	119.4	89.2	76.1	65.4	47.9	38.1	26.2	20.0	18.1	15.5
6	211.7	139.7	120.4	111.9	85.4	72.3	63.6	46.3	36.7	25.7	19.8	16.0	13.4
7	197.0	133.4	106.6	105.0	83.0	70.8	63.0	46.3	36.2	25.4	19.6	15.8	13.1
8	183.6	129.0	104.4	102.6	79.6	70.0	58.1	44.9	36.0	24.4	19.3	15.7	12.9
9	180.0	124.1	101.8	90.3	74.6	69.6	57.8	44.1	34.8	24.1	19.0	15.4	12.0
10	171.0	120.6	94.7	88.0	68.4	55.3	53.1	43.6	34.7	24.0	18.3	15.2	11.7
11	166.8	107.7	86.0	84.0	67.1	53.1	52.5	42.7	33.7	23.2	18.1	14.8	11.6
12	164.4	105.1	82.2	78.2	64.1	51.2	52.2	42.0	33.1	23.1	18.0	14.7	11.4
13	164.1	103.4	80.4	67.9	60.8	49.7	42.3	38.7	32.7	22.4	17.4	14.5	11.3
14	152.1	95.8	80.2	66.2	60.4	48.9	41.5	38.5	32.1	22.1	17.4	14.5	11.3
15	148.8	93.8	78.9	65.1	59.7	45.5	40.4	35.4	31.5	21.8	16.8	14.4	11.2
16	145.2	91.5	73.1	64.5	51.8	40.5	39.8	34.8	29.1	21.1	16.8	13.9	11.0
17	143.9	90.7	71.5	60.4	50.5	40.3	37.6	32.3	29.1	20.3	16.6	13.9	10.9
18	143.2	90.2	69.8	60.3	49.7	39.5	37.3	30.8	28.9	19.4	16.0	13.8	10.6
19	142.6	90.0	69.2	59.1	46.1	38.6	32.6	28.7	27.5	19.4	15.8	13.6	10.5
20	138.3	89.9	68.8	57.6	45.1	37.9	32.4	27.7	26.6	19.3	14.9	12.9	10.3
21	134.3	87.1	68.6	57.1	44.0	35.2	31.8	26.5	26.1	18.3	14.5	12.6	10.3
22	133.2	84.6	67.6	56.8	43.6	34.4	31.3	24.9	25.7	18.1	14.5	11.6	10.3
23	131.6	83.9	66.5	56.6	43.4	33.6	30.2	24.9	25.4	17.4	13.8	11.6	9.9
24	130.7	82.9	64.6	54.9	43.2	33.3	29.0	24.3	23.7	17.1	13.1	11.2	9.7
25	126.1	82.3	64.0	53.3	41.9	33.1	28.4	23.9	20.6	15.7	12.9	11.0	9.0
Mean	176.6	119.8	95.8	83.9	66.9	53.9	47.7	37.8	32.0	22.5	18.0	15.2	12.6
Standard Deviation	51.8	42.5	34.0	29.5	22.2	17.6	14.6	9.2	5.6	3.9	3.6	3.3	3.7
Coefficient of Skewness	1.47	1.31	1.09	0.74	0.62	0.36	0.17	-0.24	-0.23	0.27	0.83	1.27	2.41

$\bar{X}$ = mean;

K_T = Gumbel's frequency factor;

S = standard deviation

Excel optimization solver was used to calibrate the non-linear power law given by Equation (4) to obtain the model parameters C, m and a.

The calculated frequency factor for this (PDF) is as shown in Table 2.

The Gumbel's frequency factor is obtained using Equation (3):

$$K_T = -\frac{\sqrt{6}}{\pi}\left\{0.5772 + In\left[In\left(\frac{T}{T-1}\right)\right]\right\} \quad (3)$$

Where T = return period (years)

For example, Gumbel frequency factor for a 5year return period is evaluated as:

$$K_T = -\frac{\sqrt{6}}{\pi}\left\{0.5772 + In\left[In\left(\frac{10}{10-1}\right)\right]\right\} = -1.30$$

Table 2: Gumbel factor for Ibadan IDF modelling frequency

Return Period	2	5	10	25	50	100
K_T	0.16	-1.17	-1.30	-2.04	-2.59	-3.16

E. IDF Model Calibration

Sherman's (1931) IDF model is given as

$$I = \frac{CT_r{}^m}{T_d{}^a} \quad (4)$$

I = rainfall intensity, C, m and a = model parameters, T_r = return period (year) and T_d = duration (hours)

Excel optimization solver was used to calibrate the non-linear power law given by Equation (4) to obtain the model parameters C, m and a.

F. Goodness of Fit Test

The GEVT – 1 and Log-Pearson Type 3 fits the rainfall intensities with 0.757 and 0.754 significant values at 5% confidence level using the Anderson-Darling test.

III. Results and Discussion

The evaluated coefficient of determination, R^2 and Mean Square Error values for a specified return period shows that GEVT-1 best fit the rainfall maximum amounts as shown in Table 3.

Equation (1) was used in calculating the rainfall intensity values. Rainfall intensity using GEVT-1 distribution with the mean and standard deviation are obtained from Table 1. For a 10 - minute duration and 5 years return period, the probability equivalent of rainfall intensity via GEVT-1 is calculated using Equation (2) by substituting the values of X_T, $\bar{X}$, K_T and S from Table 1.

$$X_T = 119.75 + (-0.719 \times 42.54))$$

$$X_T = 150.34 \text{ mm/hr}$$

A. Specified Return Period IDF Models Calibration

[12] showed the calibration of Sherman equation IDF models for specified return periods. Table 3 shows the result for GEVT – 1 distribution which includes the coefficients of determination R^2 and mean square error (MSE) for model performance assessment.

The calibration of Equation (4) by using the Excel optimization software results in the generation of model parameters C, m and a for a given duration and return period.

These IDF models presented in Table 3 are return period specific unlike a situation where we have non-specific (general) models (see Equation 7).

Table 3: GEVT-1 calibrated IDF Models for different return periods for Ibadan.

Return Period	IDF Model ±	Coefficient of Determination (R^2)	Mean Squared Error (MSE)
2	Intensity = $\frac{4.762\,T_r{}^{6.416}}{T_d{}^{0.351}}$	0.998	3.54
5	Intensity = $\frac{2.153\,T_r{}^{3.421}}{T_d{}^{0.552}}$	0.996	10.88
10	Intensity = $\frac{1.642\,T_r{}^{2.571}}{T_d{}^{0.352}}$	0.995	23.43
25	Intensity = $\frac{1.289\,T_r{}^{1.962}}{T_d{}^{0.354}}$	0.992	48.03
50	Intensity = $\frac{1.167\,T_r{}^{1.666}}{T_d{}^{0.354}}$	0.990	72.60
100	Intensity = $\frac{1.098\,T_r{}^{1.448}}{T_d{}^{0.555}}$	0.989	102.30

± return period specific IDF models

B. Evaluation of iterative Equation Solver in Excel

50 year specified return period was used to evaluate the model parameters in Excel Solver software. It has eleven (11) iterations in the

development of the general IDF model given in Equation (7).

Table 4: Model parameters for Sherman's specific IDF model calibration

Iteration	C	M	A
1	1	1	1
2	1.1534	1.6	0.6733
3	1.1718	1.6833	0.6221
4	1.1624	1.656	0.5715
5	1.1441	1.6603	0.5719
6	1.1509	1.6855	0.5794
7	1.1509	1.6881	0.5835
8	1.1509	1.6882	0.5835
9	1.1509	1.6882	0.5835
10	1.1509	1.6882	0.5835
11	1.1509	1.6882	0.5835

Table 5: Tabular Computation of Coefficient of Determination for Ibadan

Intensity	$Intensity_{pred}$	(I - Ip)2	(I-Iavg)2
168.1278268	167.640239	0.237741934	12315.44
112.7638948	114.451644	2.848498975	3092.584
90.22885873	91.5505285	1.746811061	1094.02
79.00541384	78.1386319	0.751310956	477.5328
63.21167232	62.503541	0.501450013	36.70885
50.97729296	49.9969418	0.961088342	38.13799
45.28777369	42.6725295	6.839501956	140.781
36.30937607	34.1340018	4.732253415	434.4521
31.11422611	29.1334659	3.923411144	678.012
21.83824147	23.3040269	2.148526844	1247.124
17.41603154	19.890052	6.120777471	1579.018
14.69072999	17.5903439	8.407760939	1803.035
12.01622578	14.6154371	6.755899459	2037.318
Average = 57.153		**Sum = 45.975**	**Sum = 2497**

R^2 is computed from Equation (5) and Table 5

$$R^2 = \frac{\left(\sum_{i=1}^{n}(y-y_{avg}{}^2) - \sum_{i=1}^{n}(y-y_{pred})^2\right)}{\sum_{i=1}^{n}(y-y_{avg})^2} \qquad (5)$$

$$R^2 = \frac{24974.16-45.975}{24974.16} = 0.998$$

Calculating the Mean Square Error (MSE) using Equation (6) we have;

$$\text{MSE} = \frac{\sum_{i=1}^{n}(y-y_{pred})^2}{n} \qquad (6)$$

$$\text{MSE} = \frac{45.975}{13} = 3.54$$

A general IDF model was also developed. A total of 13 durations multiplied by 6 return period yields 78 input data point. The entire input data were taken from Table 1.

Programmed least squares equations were used to develop a general (non-specified) IDF model using Excel Optimization Solver. This resulted in Equation (7).

$$I = \frac{394.697 T_r^{\,0.175}}{T_d^{\,0.554}} \qquad (7)$$

Coefficient of determinant (R^2) = 0.990; Mean Squared Error = 57.05 mm/hr

C. Comparison of Observed and Predicted Rainfall Intensities

This model enables one to predict the intensity of rainfall of any duration and any return period. The verification of the developed model is carried out by plotting the observed and predicted intensities on the same log-log graph as shown in Figures 2.

D. Comparison of Regression Approach and Excel Optimization Solver results for model parameters, R^2 and MSE

Table 6 (an extension of Table 5) clearly shows the result from Excel Optimization Solver option is more reliable than the normal regression method, the conventional simultaneous solution using matrix i.e. Gauss elimination, inverse or determinant approach.

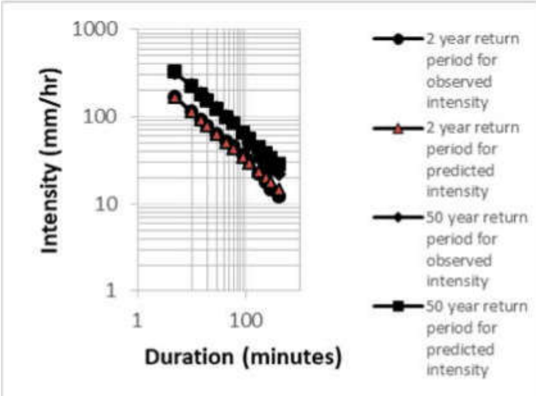

Figure 2: Log-log plot for 2 and 50 year return periods for GEVT – 1 distribution comparing observed rainfall intensity against predicted rainfall intensity.

Table 6: Results from Regression Approach and Excel Solver Optimization Approach (GEVT-1, 2 year return period)

Method	C	M	a	R^2	MSE
Regression	64.20	3.42	0.695	0.887	13.83
Excel solver	4.762	6.416	0.551	0.998	3.54

IV. Conclusion

The trend of higher intensities occurring at lower duration which is found in literature has been observed in the developed models for GEVT – 1 and Log – Pearson Type 3 distributions. The prediction of rainfall intensity with the PDFs showed a good match with observed intensity values. The log Pearson Type III model ranked as the best with respect to MSE 6.09 and R^2 0.997 in the return period specific model. The comparison of PDF and non-PDFs shows that the former has lesser MSE value than the later; 3.54 and 13.83 respectively.

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