DOI: 10.36108/ujees/3202.50.0180

UNIOSUN Journal of Engineering and Environmental Sciences, Vol. 5 No. 1. March. 2023

An Approximate Solution of Fractional Order Epidemic Model of Typhoid using the Homotopy Perturbation Method

Kolawole, M. K., Popoola, A. O., Odeyemi, K. A. and Bashiru, K. A.

Abstract: The epidemic model of typhoid is considered to be a control strategy for the undeviating transference of infectious disease. Due to its high accuracy and efficiency in solving nonlinear differential equations, the homotopy perturbation method is coupled with the Riemann-Liouville fractional integral operator I of order $\eta \in (0,1)$ to obtain the approximate analytical solution of the epidemiology model presented at different level of η. The obtained results were subjected to simulation process where the effect of the causative bacterial in the medium was studied at various level of η. The entirety of the computational procedures was conducted utilizing Maple 18, and graphical representations and tabular data were exhibited to facilitate a lucid understanding of the simulation outcomes. These findings indicate that curtailing Salmonella contamination is of paramount importance for minimizing the hazard of sickness and infection. This can be achieved through the observance of meticulous hygiene, which includes consistent hand washing with soap and water, thorough cooking of food, and appropriate storage and handling of edibles.

Keywords: Typhoid fever, Fractional differential equation, Homotopy perturbation method, Simulation.

I. Introduction

Mathematical models have been an optimal control strategy for several diseases in the last few decades [1]. Models are being increasingly used to elucidate the transmission of several diseases. These models, usually based on compartmentalized models, may be rather simple, but studying them is crucial for gaining important knowledge of the underlying aspects of how infectious diseases spread [2]. The commonly used threshold value in epidemiology is probably the basic reproduction number [3, 4]. The basic reproduction number, denoted by, is defined as the average number of secondary infections that occur when one infectious agent is introduced into a completely susceptible population. This threshold is a famous result due to the author [5] and is referred to as the "threshold phenomenon", giving a borderline between persistence and disease death. It is also called the basic reproduction ratio or basic reproductive rate [6-8]. Researchers have applied various mathematical methods, such as the homotopy perturbation and variational iteration described in [9-11], to solve epidemiology problems.

Kolawole, M.K., Popoola, A.O. and Odeyemi, K.A. (Department of Mathematical Sciences, Faculty of Basic and Applied Sciences, Osun State University, Osogbo, Nigeria)

Bashiru, K.A. (Department of Statistics, Faculty of Basic and Applied Sciences, Osun State University, Osogbo, Nigeria)

Corresponding author: mutairu.kolawole@uniosun.edu.ng

Lately, fractional calculus has been an area of interest to many researchers. Several models are now formulated with coupled fractional differential equations because it produces a greater degree of freedom for the independent variables without violating the constraints imposed on it and on the analysis and numerical simulation of the SEIR epidemic model of measles with non-integer time fractional derivatives using the Laplace-Adomian decomposition method [12]. The most common class of epidemiological mathematical models is the compartmental model. One of the popular compartmental approaches assumes that a susceptible individual first goes through a latent period (and is said to become exposed or in class E) after being infected and then recovers, in an approach known as the SEIR model. The SEIR model has been used to study two other coronavirus epidemics (SARS and MERS) since 2003 [13]. The modified homotopy perturbation method and its application to analytical solitons of the fractional-order Korteweg–de Vries equation were computed by [14]. The integer order model was adopted from [15], who researched on a mathematical model for control of measles epidemiology. A conceptual analysis of the combined effects of vaccination, therapeutic actions, and human submission to physical constraint in reducing the prevalence of COVID-19 using the homotopy perturbation method was studied by [16]. The existence of solutions to differential equations of fractional order was presented by [17]. In their research, they applied the Adomian decomposition method to obtain the result of the fractional order problem. The problem was converted to integer order to obtain the integer order result. The results obtained in the two cases were compared, and it was obvious that they were in good agreement. They concluded that the Adomian decomposition is an efficient and reliable method to solve linear problems. In this research, the mathematical model of typhoid fever described by [18] was adopted and modified to be of fractional order to study the dynamics of the disease transmission using a Caputo derivative. The existing model equation is presented in Eq. 1.

$$\left.\begin{aligned}
\frac{dS}{dt} &= \Lambda + \delta R(t) - (\mu + \lambda)S(t) \\
\frac{dC}{dt} &= \rho\lambda S(t) - (\sigma_1 + \theta + \mu + \phi)C(t) \\
\frac{dI}{dt} &= (1-\rho)\lambda S(t) + \theta C(t) - (\sigma_2 + \beta + \mu + \alpha)I(t) \\
\frac{dR}{dt} &= \beta I(t) + \phi C(t) - (\mu + \delta)R(t) \\
\frac{dB_c}{dt} &= \sigma_1 C(t) + \sigma_2 I(t) - \mu_b B_c
\end{aligned}\right\} \qquad (1)$$

Where $\lambda = \dfrac{B_c v}{k + B_c}$

In the presented model, the susceptible class is denoted by S, the carrier class by C, the infected class by I, the recovery class is represented by R, and the bacterial population is denoted by Q. λ represents the infection force, v is the rate of salmonella bacteria, and k denotes the concentration of the bacteria in sustenance.

II. Materials and Methods

A. Preliminaries and Methods

Some fundamental definitions and properties in fractional calculus are given in this section. ***Definition***: According to [16], a real function $\varphi(t), t > 0,$ is said to be in the space C_μ, $\mu \in R$ if there exist a real number $m > \mu$ such that $\varphi(t) = t^m \varphi_1(t)$. where $\varphi_1(t) \in C(0,\infty)$, and it is said to be in the space $C^n{}_\mu$ if and only if $\varphi^{(n)} \in C_\mu, n \in N.$

Definition:The Riemann-Liouville fractional integration of order $\eta \geq 0$ of a positive real function $\varphi(t) \in C_\mu$, $\mu \geq -1$ $t > 0$ is defined as Eq. (2) [16]

$$I^\eta \varphi(t) = \frac{1}{\Gamma(\eta)} \int_0^t (t-x)^{\eta-1} \varphi(x) dx \qquad (2)$$

Such that $I^0 \varphi(t) = \varphi(t).$

The following properties hold for fractional integral operator I^η for $\varphi(t) \in C_\mu$, $\mu \geq -1$ $\eta, \alpha \geq 0$ and $\beta \geq -1$:

1. $I^\eta I^\alpha \varphi(t) = I^{\eta+\alpha} \varphi(t),$
2. $I^\eta I^\alpha \varphi(t) = I^\alpha I^\beta \varphi(t),$
3. $I^\eta t^\beta = \dfrac{\Gamma(\beta+1)}{\Gamma(\eta+\beta+1)} t^{\eta+\beta}.$

Definition: The Caputo fractional derivative of a positive real function $\varphi(t)$ given as $D^\eta \varphi(t)$ is given by Eq. (3) [16]

$$D^\eta \varphi(t) = \frac{1}{\Gamma(n-\eta)} \int_0^t (t-x)^{n-\eta-1} \varphi^{(n)}(x) dx \qquad (3)$$

For $n-1 < \eta \leq n,\ n \in N,\ t > 0,\ \varphi \in C_{-1}^{n}.$

The following property holds for fractional integration of the Caputo fractional derivative. For $n-1 < \eta \leq n,\ n \in N,\ \varphi \in C_{-1}^{n}, \mu \geq -1.$

Then

$$I^\eta D^\eta \varphi(t) = \varphi(t) - \sum_{k=0}^{n-1} \varphi^{(k)}(0) \frac{t^k}{k!} \qquad (4)$$

B. Model Modification

Here, the dynamics of the model is modified to be of fractional order as fractional order derivatives give rise to greater degree of freedom and also display a realistic behavior of the effect of each parameter in the model. Thus, the modified model is presented as Eq. (5)

$$\left.\begin{aligned}
D^{\eta_1} S(t) &= \Lambda + \delta R(t) - (\mu + \lambda) S(t) \\
D^{\eta_2} C(t) &= \rho \lambda S(t) - (\sigma_1 + \theta + \mu + \phi) C(t) \\
D^{\eta_3} I(t) &= (1-\rho) \lambda S(t) + \theta C(t) - (\sigma_2 + \beta + \mu + \alpha) I(t) \\
D^{\eta_3} R(t) &= \beta I(t) + \phi C(t) - (\mu + \delta) R(t) \\
D^{\eta_1} Q(t) &= \sigma_1 C(t) + \sigma_2 I(t) - \mu_b Q(t)
\end{aligned}\right\} \quad (5)$$

Subject to the following initial conditions

$$S(0) = s_0, C(0) = c_0, I(0) = i_0, R(0) = r_0, Q(0) = q_0$$

Where $\lambda = \dfrac{Qv}{k+Q}.$

C. The Modified Homotopy Perturbation Method

To extend the theory of HPM to Eq. (5), consider a coupled fractional differential equation of order η defined as Eq. (6)

$$s_i^{(\eta_i)}(t) = \varphi(t, S_1, S_2, S_3, \cdots S_n) \qquad i \in \mathrm{N} \quad (6)$$

A homotopy can be constructed for Eq. (6) such that

$$s_i^{(\eta_i)}(t) = p(\varphi(t, S_1, S_2, S_3, \cdots S_n)) \quad i \in \mathrm{N} \qquad (7)$$

Where $p \in (0,1)$ is an embedding parameter, at $p = 0,$ Eq. (7) becomes linear such that the following Eq. (8) is obtained:

$$s_i^{(\eta_i)}(t) = 0 \qquad (8)$$

At $p = 1,$ the original equation in Eq. (6) is obtained. A series solution embedding parameter p is assumed for Eq. (6) such that:

$$s_i^{(\eta_i)}(t) = p(\varphi(t, S_1, S_2, S_3, \cdots S_n)) \quad i \in \mathrm{N}\,(9)$$

Where $p \in (0,1)$ is an embedding parameter, at $p = 0,$ Eq. (7) becomes linear such that Eq. (10) is obtained:

$$s_i^{(\eta_i)}(t) = 0 \qquad (10)$$

At $p = 1,$ the original equation in (6) is obtained. Let assume a solution series embedding the parameter p for Eq. (6) such that

$$s_i(t) = s_{i0} + ps_{i1} + p^2 s_{i2}, \cdots \qquad (11)$$

Substituting Eq. (9) into Eq. (6) and comparing the coefficients of equal powers of $p,$ the following series of equations are obtained as Eq. (12) and so on.

$$\left.\begin{aligned}
p^0: &\quad s_{i0}^{(\eta_i)}(t) = 0\\
p^1: &\quad s_{i1}^{(\eta_i)}(t) = \varphi_{i1}(t, s_{10}, s_{20}, s_{30}, \cdots s_{n0}),\\
p^2: &\quad s_{i2}^{(\eta_i)}(t) = \varphi_{i2}(t, s_{10}, s_{20}, s_{30}, \cdots s_{n0}, s_{11}, s_{21}, s_{31}, \cdots s_{n1}),\\
p^3: &\quad s_{i3}^{(\eta_i)}(t) = \varphi_{i3}(t, s_{10}, s_{20}, s_{30}, \cdots s_{n0}, s_{11}, s_{21}, s_{31}, \cdots s_{n1}, s_{12}, s_{22}, s_{32}, \cdots s_{n2}),
\end{aligned}\right\} \quad (12)$$

In turn, these series of equation in (10) can be solved by applying the Riemann-Liouville fractional integral operator I^{η} which is the inverse of D^{η} as stated in (4) to obtain the values of $s_{i1}(t), s_{i2}(t), s_{i3}(t)\ldots$.Thus, the solution of (6) is obtained by adding $s_{i1}(t), s_{i2}(t), s_{i3}(t)\ldots$ which is a truncated series is of

$$s_{iN}(t) = \sum_{k=0}^{N-1} s_{ik}(t). \qquad (13)$$

D. Application of the modified homotopy perturbation method

In this section the procedure from (6-11) will be extended to the modified model to obtain the approximate analytical solutions of the model compartments. Thus, constructing a homotopy for (5),

$$\left.\begin{aligned}
D^{\eta_1} S(t) &= p(\Lambda + \delta R(t) - (\mu + \lambda) S(t))\\
D^{\eta_2} C(t) &= P(\rho\lambda S(t) - (\sigma_1 + \theta + \mu + \phi) C(t))\\
D^{\eta_3} I(t) &= P(((1-\rho)\lambda S(t) + \theta C(t) - (\sigma_2 + \beta + \mu + \alpha) I(t))\\
D^{\eta_3} R(t) &= P(\beta I(t) + \phi C(t) - (\mu + \delta) R(t))\\
D^{\eta_1} Q(t) &= P(\sigma_1 C(t) + \sigma_2 I(t) - \mu_b Q(t))
\end{aligned}\right\} \quad (14)$$

Let the series solution embedding a parameter p for each compartment of the mode be

$$
\left.\begin{aligned}
S(t) &= s_0(t) + ps_1(t) + p^2 s_2(t) + \cdots \\
C(t) &= c_0(t) + pc_1(t) + p^2 c_2(t) + \cdots \\
I(t) &= i_0(t) + pi_1(t) + p^2 i_2(t) + \cdots \\
R(t) &= r_0(t) + pr_1(t) + p^2 r_2(t) + \cdots \\
Q(t) &= q_0(t) + pq_1(t) + p^2 q_2(t) + \cdots
\end{aligned}\right\} \quad (15)
$$

Substituting (13) into (12) and comparing equal powers of p from both sides,

Coefficients of p^0 is obtained as follows:

$$
\begin{aligned}
p^0: \quad & D^{\eta_1} i_0(t) = 0,\ D^{\eta_2} c_0(t) = 0,\ D^{\eta_3} i_0(t) = 0, \\
& D^{\eta_4} r_0(t) = 0,\ D^{\eta_5} q_0(t) = 0,
\end{aligned} \quad (16)
$$

$$
\begin{aligned}
p^1: \quad & D^{\eta_1} S_1(t) = \Lambda + \delta r_0(t) - (\mu + \lambda) s_0(t) \\
& D^{\eta_2} C_1(t) = \rho \lambda s_0(t) - (\sigma_1 + \theta + \mu + \phi) c_0(t) \\
& D^{\eta_3} I_1(t) = (1-\rho)\lambda s_0(t) + \theta c_0(t) - (\sigma_2 + \beta + \mu + \alpha) i_0(t) \quad (17) \\
& D^{\eta_3} R_1(t) = \beta i_0(t) + \phi c_0(t) - (\mu + \delta) r_0(t) \\
& D^{\eta_1} Q_1(t) = \sigma_1 c_0(t) + \sigma_2 i_0(t) - \mu_b q_0(t)
\end{aligned}
$$

$$
\begin{aligned}
p^2: \quad & D^{\eta_1} S_2(t) = \Lambda + \delta r_1(t) - (\mu + \lambda) s_1(t) \\
& D^{\eta_2} C_2(t) = \rho \lambda s_1(t) - (\sigma_1 + \theta + \mu + \phi) c_1(t) \\
& D^{\eta_3} I_2(t) = (1-\rho)\lambda s_1(t) + \theta c_1(t) - (\sigma_2 + \beta + \mu + \alpha) i_1(t) \quad (18) \\
& D^{\eta_3} R_2(t) = \beta i_1(t) + \phi s_1(t) - (\mu + \delta) r_1(t) \\
& D^{\eta_1} Q_2(t) = \sigma_1 c_1(t) + \sigma_2 i_1(t) - \mu_b q_1(t)
\end{aligned}
$$

Applying the integral operator I^η to (14) at the given initial condition,

$$
s_0(t) = s_0,\ c_0(t) = c_0,\ i_0(t) = i_0,\ r_0(t) = r_0,\ q_0(t) = q_0 \quad (19)
$$

Repeating the process for (15);

$$
\left.\begin{aligned}
s_1(t) &= (\Lambda + \delta r_0 - (\mu + \lambda) s_0) \frac{t^\eta}{\Gamma(\eta+1)} \\
c_1(t) &= (\rho \lambda s_0 - (\sigma_1 + \theta + \mu + \phi) c_0) \frac{t^\eta}{\Gamma(\eta+1)} \\
i_1(t) &= ((1-\rho)\lambda s_0 + \theta c_0 - (\sigma_2 + \beta + \mu + \alpha) i_0) \frac{t^\eta}{\Gamma(\eta+1)} \\
r_1(t) &= (\beta i_0 + \phi c_0 - (\mu + \delta) r_0) \frac{t^\eta}{\Gamma(\eta+1)} \\
q_1(t) &= (\sigma_1 c_0 + \sigma_2 i_0 - \mu_b q_0) \frac{t^\eta}{\Gamma(\eta+1)}
\end{aligned}\right\} \quad (20)
$$

Also doing the same for (16) yields the compartmental results of the second iteration. Such that

$$
s_2(t) = (\delta\theta c_0 + \delta\beta i_0 - \delta^2 r_0 - 2\delta\mu r_0 - \mu\Lambda + \mu^2 s_0 + 2\mu\lambda s_0 - \lambda\Lambda - \lambda\delta c_0 + \lambda^2 s_0) \frac{t^{2\eta}}{\Gamma(2\eta+1)} \quad (21)
$$

$$
\begin{aligned}
c_2(t) = {} & (2\sigma_1 \phi c_0 - \sigma_1 \rho\lambda s_0 + 2\sigma_1 \mu c_0 + 2\sigma_1 \theta c_0 + \sigma_1^2 c_0 + \rho\lambda\Lambda + \rho\lambda\delta r_0 - 2\rho\lambda\mu s_0 - \rho\lambda^2 s_0 + 2\mu\phi c_0 \\
& + \mu^2 c_0 + 2\mu\theta c_0 + 2\theta\phi c_0 - \theta\rho\lambda s_0 + \theta^2 c_0 - \phi\rho\lambda s_0) \frac{t^{2\eta}}{\Gamma(2\eta+1)}
\end{aligned} \quad (22)
$$

$$
\begin{aligned}
i_2(t) = {} & (-\theta^2 c_0 + 2\sigma_2 \alpha i_0 + 2\alpha\beta i_0 + 2\sigma_2 \mu i_0 - \sigma_2 \theta c_0 - \sigma_2 \lambda s_0 + \sigma_2 \rho\lambda s_0 + \alpha\rho s_0 + \beta\rho\lambda s_0 + 2\mu\beta i_0 \\
& - \beta\lambda s_0 - \beta\theta c_0 + 2\alpha\mu i_0 - \alpha\theta c_0 - \rho\lambda\delta r_0 + 2\rho\lambda\mu s_0 - \alpha\lambda s_0 + \theta\rho\lambda s_0 + 2\sigma_2 \beta i_0 - \sigma_1 \theta c_0 \\
& - \rho\lambda\Lambda + \rho\lambda^2 s_0 - 2\mu\theta c_0 - \theta\phi c_0 + \sigma_2^2 i_0 + \alpha^2 i_0 + \mu^2 i_0 + \beta^2 i_0 + \lambda\Lambda - \lambda^2 s_0 - 2\mu\lambda s_0 + \lambda\delta r_0) \\
& \frac{t^{2\eta}}{\Gamma(2\eta+1)}
\end{aligned} \quad (23)
$$

$$
\begin{aligned}
r_2(t) = {} & (-\theta\phi c_0 + \theta\rho\lambda s_0 - 2\mu\theta c_0 - \theta^2 c_0 - \sigma_1 \theta c_0 - \beta^2 i_0 + \beta\lambda s_0 - \alpha\beta i_0 - 2\mu\beta i_0 - \beta\rho\lambda s_0 - \sigma_2 \beta i_0 \\
& + \theta\beta c_0 - \delta\theta c_0 - \delta\beta i_0 + \delta^2 r_0 + 2\delta\mu r_0 + \mu^2 r_0) \frac{t^{2\eta}}{\Gamma(2\eta+1)}
\end{aligned} \quad (24)
$$

$$
\begin{aligned}
q_2(t) = {} & -(\sigma_1 \phi c_0 - \sigma_1 \rho s_0 + \sigma_1 \mu c_0 + \sigma_1 \theta c_0 + \sigma_1^2 c_0 + \sigma_2 \beta i_0 - \sigma_2 \lambda s_0 + \sigma_2 \alpha i_0 + \sigma_2 \mu i_0 + \sigma_2 \rho\lambda s_0 \\
& + \sigma_2^2 i_0 - \sigma_2 \theta c_0 + \mu_b \sigma_2 i_0 + \mu_b \sigma_1 c_0 - \mu_b^2 q_0) \frac{t^{2\eta}}{\Gamma(2\eta+1)}
\end{aligned} \quad (25)
$$

The approximate solutions of each class are obtained by adding the three iterative solutions. Such that:

$$\left.\begin{array}{l} S(t)=\sum_{n=0}^{\infty} s_n(t), \\ C(t)=\sum_{n=0}^{\infty} c_n(t), \\ I(t)=\sum_{n=0}^{\infty} i_n(t), \\ R(t)=\sum_{n=0}^{\infty} r_n(t), \\ Q(t)=\sum_{n=0}^{\infty} q_n(t)\ldots \end{array}\right\} \qquad (26)$$

The following values are suggested as initial population parameter for the classes of the epidemic model $S(0)=500,\ C(0)=400,\ I(0)=300,\ R(0)=350,\ Q(0)=100$ all other parameters are adopted from [15]. The approximate results in equation (19-23) are evaluated using the prescribed parameters and the following results are consequently obtained:

$$S(t)=500+\frac{87.9574000t^{\eta}}{\Gamma(\eta+1)}-\frac{2.181581239t^{2\eta}}{\Gamma(2\eta+1)}$$

$$C(t)=400+\frac{(-89.9973-400\sigma_1)t^{\eta}}{\Gamma(\eta+1)}-\frac{(20.24986747+179.9973\sigma_1+400\sigma_1^2)t^{2\eta}}{\Gamma(2\eta+1)}$$

$$I(t)=300-\frac{183.6037000t^{\eta}}{\Gamma(\eta+1)}-\frac{(143.3342194-80.0\sigma_1)t^{2\eta}}{\Gamma(2\eta+1)}$$

$$R(t)=350-\frac{(8.241400)t^{\eta}}{\Gamma(\eta+1)}-\frac{(143.3342194-80.0\sigma_1)t^{2\eta}}{\Gamma(2\eta+1)}$$

.

III. Results and Discussion

The analysis of the effect of σ_1 which is the typhoid causative bacteria in the model was carried out by varying it on the interval $0.3 \le \sigma_1 \le 0.9$ at $0 < \eta \le 1$. Graphical interpretations of the results obtained are presented to give a better exposition of its effects in the medium.

Carrier Class

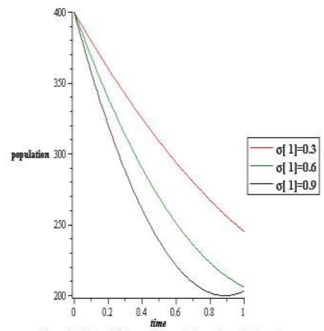

Figure 1:- Effect of σ[1] on carrier population at time t of order η=1

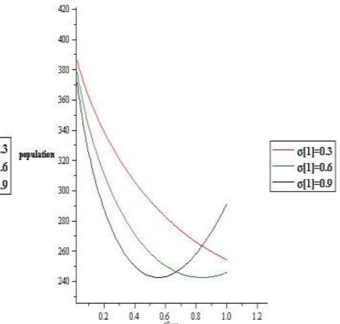

Figure 2:- Effect of σ[1] on carrier population at time t of order η=0.75

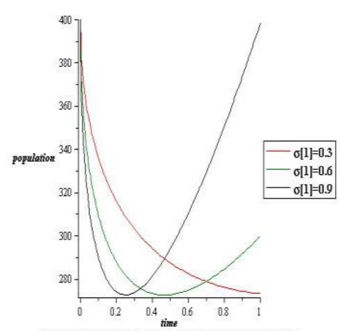

Figure 3:-Effect of σ[1] on carrier population at time t of order η=0.5

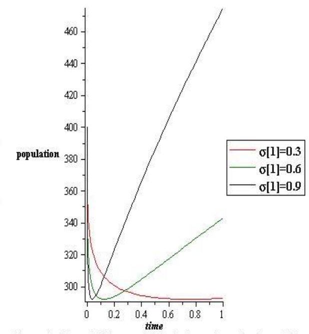

Figure 4:- Effect of σ[1] on carrier population at time t of order η=0.25

Infected Class

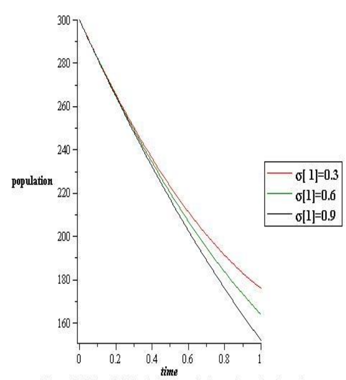

Figure 5:- Effect of σ[1] on infected population at time t of order η=1

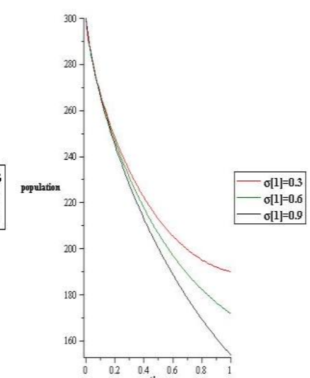

Figure 6:-Effect of σ[1] on infected population at time t of order η=0.75

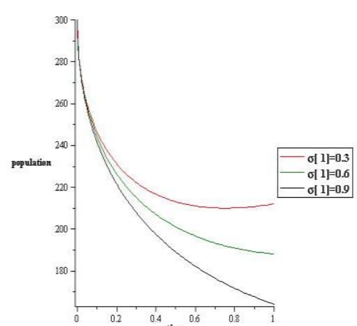

Figure 7:-Effect of σ[1] on infected population at time t of order η=0.5

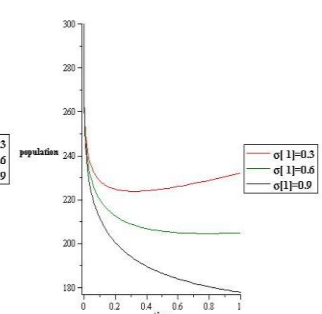

Figure 8 :-Effect of σ[1] on infected population at time t of order η=0.25

Recovered Class

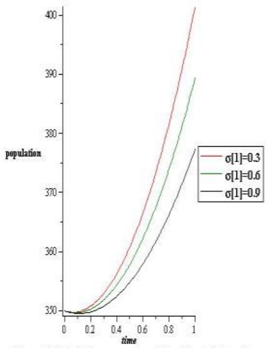

Figure 9:-Effect of σ[1] on recovered population at time t of order η=1

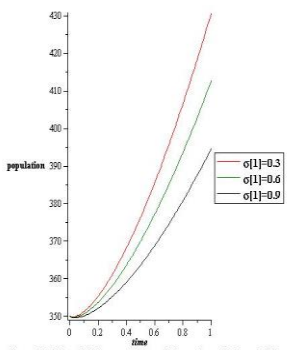

Figure 10:- Effect of σ[1] on recovered population at time t of order η=0.75

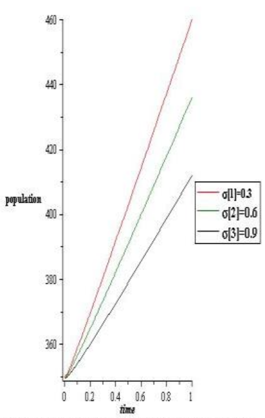

Figure 11:- Effect of σ[1] on recovered population at time t of order η=0.5

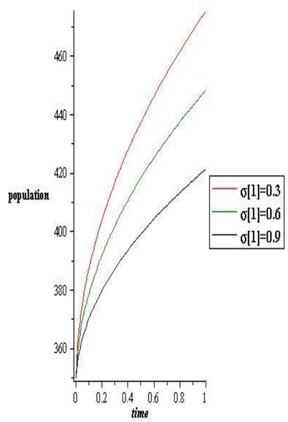

Figure 12:-Effect of σ[1] on recovered population at time t of order η=0.25

The presented graphs in Figures 1-4 and Figures 5-8 reveals that the population of the carrier and infected compartments decrease drastically over time as σ_1 increases from 0 to 0.9. It is obvious that the impact of σ_1 in the classes is dependent on η. Although the population of the carrier and infected classes reduces as σ_1 increases, it was discovered that the decrement rate becomes lower as η decreases from $1\ to\ 0.25$. In the recovery class, due to the influence of parameters such as vaccination and immunity, there is an increment in the population of the class. It was seen that more people tend to be recovered as σ_1 decreases from $0.9\ to\ 0.3$.

A. Conclusions

The objective of this study was to use the homotopy perturbation method to obtain a numerical solution for a fractional-order typhoid model. This method proved to be highly effective in generating accurate results for the model. The numerical output was then simulated to investigate the impact of typhoid-causing bacteria in the surrounding medium. The accompanying graphs were carefully analyzed to uncover the experimental findings. However, the study acknowledges that further research is needed to address the widespread prevalence of this epidemic and to develop appropriate measures for its eradication and control. Sensitization measures can play a crucial role in this regard by raising awareness and promoting preventive measures to curb the spread of typhoid.

References

[1] Heathcote H. W. “A thousand and one epidemic models”. *Frontiers in Theoretical Biology*”, vol. 100**,** no. 2, 1994, pp. 504–515.

[2] Momoh, A.A., Ibrahim, M.O., Uwanta, I.J., and Manga, S.B. “Mathematical model for control of measles epidemiology”. *International Journal of Pure and Applied Mathematics*, vol. 87, no. 5, 2013, pp. 707-718.

[3] Yunus, A. O., Olayiwola, M. O., Adedokun, K. A., Adedeji, J. A., and Alaje, A. I. “Mathematical analysis of fractional-order caputo’s derivative of coronavirus disease model via laplace adomian decomposition method”. *Beni-Suef University Journal of Basic and Applied Sciences*, vol. 144, no. 11, 2022, pp. 18-28.

[4] Das, K., Kumar, G. R., Reddy, K. M., and Lakshminarayan, K. “Sensitivity and elasticity

analysis of covel corona virus transmission model". *A mathematical approach sensors International*, vol. 2, https://doi.org/10.1016/j.sintl.2021.100088

[5] Castillo, C., Feng, Z., and Huang, W. "On the computation of R_0 and its role in global stability in mathematical approaches for emerging and reemerging infectious diseases": *An Introduction Springer*, Vol. 125, 2002, pp. 229–250.

[6] Farman, J., "Variational iteration method, a kind of non-linear analytical technique", *International Journal of Nonlinear Mechanics*"., vol. 34 no. 4, 2018, pp. 699-708.

[7] Kolawole, M.K., Ogundeji, O.D. and Popoola, A.O. "comparison of saturation term and disease induced in SIR epidemic model", *International Journal of Research and Innovation in Applied Science*, vol. 5, No. 1, 2020, pp. 2454-6194.

[8] He, J.H, "Homotopy perturbation technique computation methods". *Applied Mechanical Engineering*, vol. 178, no. 6, 2017, pp. 257-262.

[9] Olayiwola, M. O., Gbolagade, A. W. and Adesanya, A. O. "Solving variable coefficient fourth-order parabolic equation by modified initial guess variational iteration method". *Journal of the Nigerian Association of Mathematical Physics*', vol. 16, 2010, pp. 205-210.

[10] Olayiwola, M. O., Gbolagade, A. W., and Adesanya, A. O. "An efficient algorithm for solving the telegraph equation". *Journal of the Nigerian Association of Mathematical Physics*, vol. 16, 2010, pp. 199-204.

[11] He, J.H. A "Coupling method of a homotopy technique and a perturbation technique for non linear problems". *International journal of non linear mechanics*, vol. 35, no. 1, 2000, pp 37-43.

[12] Kolawole, M.K., Ayoola, T.A. and Abdulrasaq, R. "Modelling typhoid fever with general knowledge, vaccination and treatment for susceptible individual". *International Journal of Research and Innovation in Applied Science*, vol. 5, no. 3, 2020, pp. 2454-2469.

[13] Olayiwola, M.O., Gbolagade, A.W., Ayeni, R.O. and Mustapha, A.R. "On the existence of solution of differential equation of fractional order". *Medwell Journals*, vol. 2, no. 5, 2008, pp. 157-159.

[14] Muhammad F.A. and Muhammad S.O, (2018). "Analysis and numerical solution of seir epidemic model of measles with a non-integer time fractional derivatives using the laplace adomian decomposition method". *Ain Shans Engineering Journal*, vol. 9, 2018, pp 3391-3397.

[15] Tilahun, G.T., Makinde, O. and Malonza, D. "Modelling and optimal control of typhoid fever disease with optimal control strategy". *Journal of Computational and Mathematical methods in medicine*, vol. 16, 2017, pp. 16-24.

[16] Nawaz. Y. "Variational iteration method and homotopy perturbation method for fourth order fractional integro-differential equations". *Journal of Coputational Mathematics and Applied Sciences*, vol. 61, 2011, pp. 2330-2341.

[17] Kolawole, M.K., Olayiwola, M.O. and Odeyemi, K. A., "Conceptual analysis of the combined effects of vaccination, therapeutic

actions, and human subjection to physical constraint in reducing the prevalence of covid-19 using the homotopy perturbation method". *Beni-Suef University Journal of Basic Applied and Science*, vol. 12, no. 10, 2023, https://doi.org/10.1186/s43088-023-00343-2

[18] Alaje, A.I., Olayiwola, M.O., Adedokun, K.A. "Modified homotopy perturbation method and its application to analytical solitons of fractional-order korteweg–de vries equation". *Beni-Suef University Journal of Basic Applied and Science*, vol. 11, no. 139.2022 https://doi.org/10.1186/s43088-022-00317-w