

Conceptual Investigation of the Disease Transmission Coefficient in Seir Epidemic Model Using Laplace Adomian Decomposition Method (LADM)

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Abstract: In this paper, the impact of disease transmission coefficient on a SEIR epidemic model using Laplace Adomian Decomposition Method (LADM) towards disease eradication is presented. Numerical Simulations which show the effect of transmission coefficient are shown with the use of Maple 18 and the results discussed extensively. The simulation results show that disease transmission coefficient plays vital role in disease eradication.

Keywords: SEIR Epidemic model, Disease Transmission coefficient, Laplace-Adomian Decomposition, Maple 18

I. Introduction

Computation and approximation techniques such as the variational iteration method proposed by [1] have been used by several researchers to obtain the solution of different mathematical models. This method was successfully applied by [2] to replicate the effect of saturation term on a coupled SEIRS epidemic model. The Laplace Adomian decomposition method (LADM) is another good approximation technique that was applied by [3] to carry out the analysis and simulation on a mathematical model of measles. The variational iteration method was applied by [4] on a Susceptible Exposed Infected Recovered epidemic model having a saturated incidence rate to simulate the effect of saturation term in it.

[5] Applied the LADM on a fractional order model of smoking to obtain its approximate solution. An investigation of the effect of disease transmission coefficient in a SEIR model was carried out in 2019 by [6]. The

Homotopy Analysis Method (HAM) was used to provide an accurate result of a tuberculosis model by [10] the computation software Maple 15 was used to do the computational works on the SEIR model and their result signifies the efficiency of the (HAM) in solving non-linear coupled differential equations. In this paper, the impact of the disease transmission coefficient in a deterministic SEIR epidemic model adopted from the research of [6] is investigated by applying the LADM. The coupled ordinary differential equation that illustrates the model is presented as equation 1:

$$\begin{aligned}\dot{S}(t) &= \Lambda - \frac{\beta S(t)I(t)}{1 + m_1 S(t) + m_2 I(t)} - \mu S(t) \\ \dot{E}(t) &= \frac{\beta S(t)I(t)}{1 + m_1 S + m_2 I} - (\mu + \varepsilon)E(t) \\ \dot{I}(t) &= \varepsilon E(t) - (\gamma + \mu)I(t) \\ \dot{R}(t) &= \gamma I(t) - (\mu + \delta)R(t)\end{aligned}\quad (1)$$

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For easy application of the LADM on the model, the incidence rate $\frac{1}{1 + m_1 S(t) + m_2 I(t)}$ is denoted by λ such that we have

$$\begin{aligned}
\dot{S}(t) &= \Lambda - (\beta I(t)\lambda + \mu)S(t) \\
\dot{E}(t) &= \beta S(t)I(t)\lambda - (\mu + \varepsilon)E(t) \\
\dot{I}(t) &= \varepsilon E(t) - (\gamma + \mu)I(t)
\end{aligned}
\quad (2)$$

$$\dot{R}(t) = \gamma I(t) - (\mu + \delta)R(t)$$

Subject to the following initial conditions

$$S(0) = s_0, E(0) = e_0, I(0) = i_0, R(0) = r_0$$

Parameters and Descriptions

The susceptible, exposed, infected and recovered compartments are denoted with S-E-I-R respectively. β stands for the coefficient of disease transmission, μ and ε respectively represent the mortality and recuperation rate, γ represents the rate of losing immunity, the incident rate is denoted by λ and the birth rate Λ represents the total population of the model.

II. Materials and Methods

A. Laplace Adomian Decomposition Method

In this section, iterative solution of the described model is obtained by using the (LADM) introduced by [7]. An application of this method was extended by [8] to obtain the solution of IVP. This method was also applied to solve an HIV infection model by [9]. Here, some applicable definitions applicable in the model are defined.

Definitions:

Let $\gamma(t)$ be a function continuous for all positive real number $t \geq 0$. The Laplace Transform of the function is

$$\gamma(s) = \int_0^{\infty} e^{-st} \gamma(t) dt$$

- 2 For a differentiable function $\varphi(t)$ of order Ω the Laplace transform is given by

$$\ell[\gamma^\Omega(t)] = \left\{ \begin{aligned} &\pi^\Omega \ell[\gamma(t)] - \pi^{\Omega-1} \gamma(0) \\ &- \pi^{\Omega-2} \gamma'(0) - \pi^{\Omega-3} \gamma''(0) \dots \end{aligned} \right\}$$

- 3 The Laplace transform inverse of $\frac{\gamma(s)}{s}$ is

$$\ell^{-1} \frac{\gamma(s)}{s} = \int_0^t \gamma(t) dt$$

B. Application

To apply the technique mentioned, we start by taking the Laplace operator of each sides of (1)

$$\begin{aligned}
\ell \left[\dot{S}(t) \right] &= \ell[\Lambda] - \beta \lambda \ell[S(t)I(t)] - \mu \ell[S(t)] \\
\ell \left[\dot{E}(t) \right] &= \beta \lambda \ell[S(t)I(t)] - (\mu + \varepsilon) \ell[E(t)] \\
\ell \left[\dot{I}(t) \right] &= \varepsilon \ell[E(t)] - (\gamma + \mu) \ell[I(t)] \\
\ell \left[\dot{R}(t) \right] &= \gamma \ell[I(t)] - \mu \ell[R(t)]
\end{aligned}
\quad (3)$$

Following definition (ii) which is the inverse of Laplace transform, equation (4) is obtained:

$$\begin{aligned}
\pi \ell[S(t)] - S(0) &= \frac{\Lambda}{\pi} - \beta \lambda \ell[S(t)I(t)] - \mu \ell[S(t)] \\
\pi \ell[E(t)] - E(0) &= \beta \lambda \ell[S(t)I(t)] - (\mu + \varepsilon) \ell[E(t)] \\
\pi \ell[I(t)] - I(0) &= \varepsilon \ell[E(t)] - (\gamma + \mu) \ell[I(t)] \\
\pi \ell[R(t)] - R(0) &= \gamma \ell[I(t)] - \mu \ell[R(t)]
\end{aligned}
\quad (4)$$

Simplifying equation (4) by appropriately substituting the initial conditions and dividing both sides by π , (5) is obtained

$$\begin{aligned}
\ell[S(t)] &= \frac{s_0}{\pi} + \frac{\Lambda}{\pi^2} - \frac{\beta \lambda}{\pi} \ell[S(t)I(t)] - \frac{\mu}{\pi} \ell[S(t)] \\
\ell[E(t)] &= \frac{e_0}{\pi} + \frac{\beta \lambda}{\pi} \ell[S(t)I(t)] - \frac{(\mu + \varepsilon)}{\pi} \ell[E(t)] \\
\ell[I(t)] &= \frac{i_0}{\pi} + \frac{\varepsilon}{\pi} \ell[E(t)] - \frac{(\gamma + \mu)}{\pi} \ell[I(t)] \\
\ell[R(t)] &= \frac{r_0}{\pi} + \frac{\gamma}{\pi} \ell[I(t)] - \frac{\mu}{\pi} \ell[R(t)]
\end{aligned}
\quad (5)$$

Or

$$\begin{aligned}
\ell[S(t)] &= \frac{s_0}{\pi} + \frac{\Lambda}{\pi^2} - \frac{\beta\lambda}{\pi} \ell[\omega(t)] - \frac{\mu}{\pi} \ell[S(t)] \\
\ell[E(t)] &= \frac{e_0}{\pi} + \frac{\beta\lambda}{\pi} \ell[\omega(t)] - \frac{(\mu+\varepsilon)}{\pi} \ell[E(t)] \\
\ell[I(t)] &= \frac{i_0}{\pi} + \frac{\varepsilon}{\pi} \ell[E(t)] - \frac{(\gamma+\mu)}{\pi} \ell[I(t)] \\
\ell[R(t)] &= \frac{r_0}{\pi} + \frac{\gamma}{\pi} \ell[I(t)] - \frac{\mu}{\pi} \ell[R(t)]
\end{aligned} \quad (6)$$

where $\omega(t) = S(t)I(t)$ represent the nonlinear term. The solution of each compartment of (6) which has a nonlinear term is represented as an infinite series given by (7)

$$\left. \begin{aligned} S(t) &= \sum_{i=0}^{\infty} S_i(t), \quad E(t) = \sum_{i=0}^{\infty} E_i(t), \\ I(t) &= \sum_{i=0}^{\infty} I_i(t), \quad R(t) = \sum_{i=0}^{\infty} R_i(t) \end{aligned} \right\} \quad (7)$$

The infinite series of the nonlinear term is

$$\omega(t) = \sum_{n=0}^{\infty} \omega_n(t), \quad (8)$$

The first four Adomian polynomial of the nonlinear term $\omega_n(t)$ is given by;

$$\begin{aligned}
\omega_0 &= S_0 I_0 \\
\omega_1 &= S_0 I_1 + S_1 I_0 \\
\omega_2 &= S_2 I_0 + S_1 I_1 + S_0 I_2 \\
\omega_3 &= S_3 I_0 + S_2 I_1 + S_1 I_2 + S_0 I_3 \\
\omega_4 &= S_4 I_0 + S_3 I_1 + S_2 I_2 + S_1 I_3 + S_0 I_4
\end{aligned} \quad (9)$$

Substituting (7) and (8) into (6), the following equations are obtained

$$\left. \begin{aligned} \ell\left[\sum_{i=0}^{\infty} S_i(t)\right] &= \frac{s_0}{\pi} + \frac{\Lambda}{\pi^2} - \frac{\beta\lambda}{\pi} \ell\left[\sum_{i=0}^{\infty} \omega_n(t)\right] - \frac{\mu}{\pi} \ell\left[\sum_{i=0}^{\infty} S_i(t)\right] \\ \ell\left[\sum_{i=0}^{\infty} E_i(t)\right] &= \frac{e_0}{\pi} + \frac{\beta\lambda}{\pi} \ell\left[\sum_{i=0}^{\infty} \omega_i(t)\right] - \frac{(\mu+\varepsilon)}{\pi} \ell\left[\sum_{i=0}^{\infty} E_i(t)\right] \\ \ell\left[\sum_{i=0}^{\infty} I_i(t)\right] &= \frac{i_0}{\pi} + \frac{\varepsilon}{\pi} \ell\left[\sum_{i=0}^{\infty} E_i(t)\right] - \frac{(\gamma+\mu)}{\pi} \ell\left[\sum_{i=0}^{\infty} I_i(t)\right] \\ \ell\left[\sum_{i=0}^{\infty} R_i(t)\right] &= \frac{r_0}{\pi} + \frac{\gamma}{\pi} \ell\left[\sum_{i=0}^{\infty} I_i(t)\right] - \frac{\mu}{\pi} \ell\left[\sum_{i=0}^{\infty} R_i(t)\right] \end{aligned} \right\} \quad (10)$$

The iterative terms obtained by matching the two sides of (10) are:

Susceptible compartment:

$$\begin{aligned}
\ell[S_0(t)] &= \frac{s_0}{\pi} + \frac{\Lambda}{\pi^2} \\
\ell[S_1(t)] &= -\frac{\beta\lambda}{\pi} \ell[\omega_0(t)] - \frac{\mu}{\pi} \ell[S_0(t)] \\
\ell[S_2(t)] &= -\frac{\beta\lambda}{\pi} \ell[\omega_1(t)] - \frac{\mu}{\pi} \ell[S_1(t)] \\
&\vdots \\
\ell[S_{n+1}(t)] &= -\frac{\beta\lambda}{\pi} \ell[\omega_n(t)] - \frac{\mu}{\pi} \ell[S_n(t)]
\end{aligned}$$

Exposed compartment:

$$\begin{aligned}
\ell[E_0(t)] &= \frac{e_0}{\pi} \\
\ell[E_1(t)] &= \frac{\beta\lambda}{\pi} \ell[\omega_0(t)] - \frac{(\mu+\varepsilon)}{\pi} \ell[E_0(t)] \\
\ell[E_2(t)] &= \frac{\beta\lambda}{\pi} \ell[\omega_1(t)] - \frac{(\mu+\varepsilon)}{\pi} \ell[E_1(t)] \\
&\vdots \\
\ell[E_{n+1}(t)] &= \frac{\beta\lambda}{\pi} \ell[\omega_n(t)] - \frac{(\mu+\varepsilon)}{\pi} \ell[E_n(t)]
\end{aligned}$$

Infected compartment:

$$\begin{aligned}
\ell[I_0(t)] &= \frac{i_0}{\pi} \\
\ell[I_1(t)] &= \frac{\varepsilon}{\pi} \ell[E_0(t)] - \frac{(\gamma+\mu)}{\pi} \ell[I_0(t)] \\
\ell[I_2(t)] &= \frac{\varepsilon}{\pi} \ell[E_1(t)] - \frac{(\gamma+\mu)}{\pi} \ell[I_1(t)] \\
&\vdots \\
\ell[I_{n+1}(t)] &= \frac{\varepsilon}{\pi} \ell[E_n(t)] - \frac{(\gamma+\mu)}{\pi} \ell[I_n(t)]
\end{aligned}$$

Recovered compartment

$$\begin{aligned}
\ell[R_0(t)] &= \frac{r_0}{\pi} \\
\ell[R_1(t)] &= \frac{\gamma}{\pi} \ell[I_0(t)] - \frac{\mu}{\pi} \ell[R_0(t)] \\
\ell[R_2(t)] &= \frac{\gamma}{\pi} \ell[I_1(t)] - \frac{\mu}{\pi} \ell[R_1(t)] \\
&\vdots \\
\ell[R_{n+1}(t)] &= \frac{\gamma}{\pi} \ell[I_n(t)] - \frac{\mu}{\pi} \ell[R_n(t)]
\end{aligned}$$

Following definition (ii) the inverse transform of the Laplace operator was applied to each classes of the model and the initial approximation of each class is obtained as

$$\begin{aligned}
S_0(t) &= s_0 + \Lambda t, \\
E_0(t) &= e_0, \\
I_0(t) &= i_0, \\
R_0(t) &= r_0.
\end{aligned} \quad (11)$$

The first approximation is similarly obtained

$$\begin{aligned}
S_1(t) &= -(\mu s_0 + \beta \lambda s_0 i_0)t - (\mu \Lambda + \beta \lambda \Lambda i_0) \frac{t^2}{2} \\
\text{as: } E_1(t) &= (-\varepsilon e_0 + \beta \lambda s_0 i_0 - \mu e_0)t + (\beta \lambda \Lambda i_0) \frac{t^2}{2} \\
I_1(t) &= (\varepsilon e_0 - i_0 \gamma - i_0 \mu)t \\
R_1(t) &= (i_0 \gamma - r_0 \mu)t
\end{aligned} \quad (12)$$

The second approximation is obtained as

$$\begin{aligned}
S_2(t) &= \left\{ \begin{aligned} &(\lambda^2 \beta^2 s_0 i_0^2 + \lambda \beta s_0 i_0 \gamma \\ &+ \lambda \beta s_0 e_0 \varepsilon + 3 \lambda \beta s_0 i_0 \mu) \frac{t^2}{2} \\ &+ (4 \lambda \beta \Lambda i_0 \mu + \lambda \beta s_0 i_0 \gamma \\ &- 2 \lambda \beta \Lambda e_0 \varepsilon + \mu^2 \Lambda + 2 \lambda \beta \Lambda i_0 \gamma) \frac{t^3}{6} \end{aligned} \right\}
\end{aligned}$$

$$\begin{aligned}
E_2(t) &= \left\{ \begin{aligned} &(-\lambda^2 \beta^2 s_0 i_0^2 - \lambda \beta s_0 i_0 \gamma + \lambda \beta s_0 e_0 \varepsilon \\ &- 3 \lambda \beta s_0 i_0 \mu - \varepsilon^2 e_0 + \mu^2 e_0 + 2 \mu \varepsilon e_0 \\ &- \varepsilon \lambda \beta i_0 s_0) \frac{t^2}{2} + (-\lambda^2 \beta^2 i_0^2 \Lambda - 4 \lambda \beta \Lambda i_0 \mu \\ &- 2 \lambda \beta \Lambda i_0 \gamma - \lambda \beta i_0 \Lambda \varepsilon + 2 \varepsilon e_0 \lambda \beta \Lambda) \frac{t^3}{6} \end{aligned} \right\} \\
I_2(t) &= \left\{ \begin{aligned} &-\varepsilon^2 e_0 + \varepsilon \lambda \beta s_0 i_0 + 2 \mu i_0 \gamma - 2 \mu \varepsilon e_0 \\ &+ i_0 \mu^2 - \gamma \varepsilon e_0 + i_0 \gamma^2) \frac{t^2}{2} + (\varepsilon \lambda \beta i_0 \Lambda) \frac{t^3}{6} \end{aligned} \right\}
\end{aligned} \quad (13)$$

$$R_2(t) = (2 \gamma \varepsilon e_0 - \mu i_0 \gamma - 2 i_0 \gamma^2 + 2 \mu^2 r_0) \frac{t^2}{2}$$

The third approximations of each of the compartments are obtained using the following algorithm.

```

S[3](t):=collect(expand(inttrans[:invlaplace](((lambda*beta/alpha)*inttrans[:laplace]((S[2](t)*Iota[0](t)+Iota[1](t)*S[1](t)+S[0](t)*Iota[2](t)),t,alpha)-(mu/alpha)*inttrans[:laplace](S[2](t),t,alpha)),alpha,t)),t);

```

```

E[3](t):=collect(expand(inttrans[:invlaplace](((lambda*beta/alpha)*inttrans[:laplace]((S[2](t)*Iota[0](t)+Iota[1](t)*S[1](t)+S[0](t)*Iota[2](t)),t,alpha)-((mu+epsilon)/alpha)*inttrans[:laplace](E[2](t),t,alpha)),alpha,t)),t);

```

```

I[3](t):=collect(expand(inttrans[:invlaplace](((epsilon/alpha)*inttrans[:laplace](E[2](t),t,alpha)-((mu+gamma)/alpha)*inttrans[:laplace](Iota[2](t),t,alpha)),alpha,t)),t);

```

```

R[3](t):=inttrans[:invlaplace](((gamma/alpha)*inttrans[:laplace](Iota[2](t),t,alpha)-((mu/alpha)*inttrans[:laplace](R[2](t),t,alpha)),alpha,t);

```

Such that the approximate results of each of the class is

$$\begin{aligned}
S(t) &= \sum_{i=0}^3 S_i(t) = S_0(t) + S_1(t) + S_2(t) + S_3(t), \\
E(t) &= \sum_{i=0}^3 E_i(t) = E_0(t) + E_1(t) + E_2(t) + E_3(t), \\
I(t) &= \sum_{i=0}^3 I_i(t) = I_0(t) + I_1(t) + I_2(t) + I_3(t), \quad (14) \\
R(t) &= \sum_{i=0}^3 R_i(t) = R_0(t) + R_1(t) + R_2(t) + R_3(t).
\end{aligned}$$

Maple 18 is employed for the evaluation of the results obtained in (14) and their respective parametric values presented below:

Following the evaluation, the results obtained for each class are accordingly presented

Table 1: Results Evaluation and Their Respective Parametric Values

Parameter	Value	Source
s_0	13	Assumed
i_0	8	Assumed
e_0	11	Assumed
r_0	9	Assumed
$\in$	0.25	[4]
μ	0.3	[4]
γ	0.1	[4]
λ	0.16	Estimated

$$\begin{aligned}
S(t) &= \left. \begin{aligned} &13 + (37.1 - 16.64\beta)t \\ &+ (-5.56500000 - 20.78000000\beta \\ &+ 10.64960000\beta^2)t^2 + (-5.56500000 \\ &+ 5.80473333\beta + 5.95968000\beta^2 \\ &- 4.54382933\beta^3)t^3 + (0.0461250000 \\ &- 0.31775000\beta - 7.85450666\beta^2 \\ &+ 3.58263467\beta^3)t^4 - 2.86890667\beta^2t^5. \end{aligned} \right\}
\end{aligned}$$

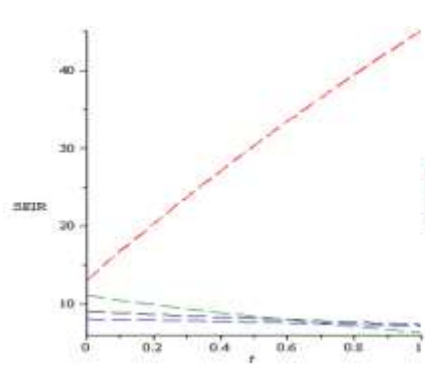
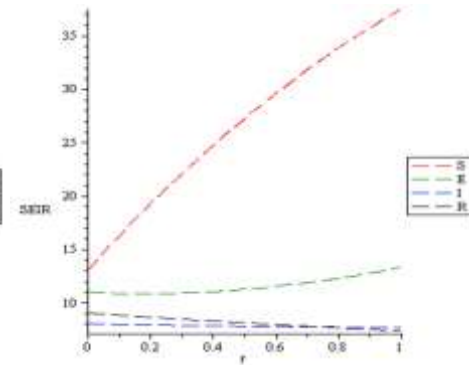
$$\begin{aligned}
E(t) &= \left. \begin{aligned} &11 + (6.05 - 16.64\beta)t \\ &+ (1.66375000 + 18.70000000\beta \\ &- 10.64960000\beta^2)t^2 \\ &+ (-0.305028333 - 7.15506666\beta \\ &+ 5.07221333\beta^2 - 4.54382933\beta^3)t^3 \\ &+ (0.37241667\beta - 8.55424000\beta^2 \\ &+ 3.58263467\beta^3)t^4 - 2.86890667\beta^2t^5 \end{aligned} \right\}
\end{aligned}$$

$$\begin{aligned}
I(t) &= \left. \begin{aligned} &8 - 0.45t + (-0.666250000 + 2.08000000\beta)t^2 \\ &+ (0.227479167 + 1.28100000\beta \\ &- 0.88746667\beta^2)t^3 + \\ &(0.744833337\beta - 0.69973333\beta^2)t^4 \end{aligned} \right\}
\end{aligned}$$

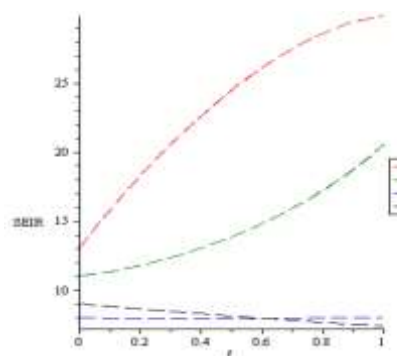
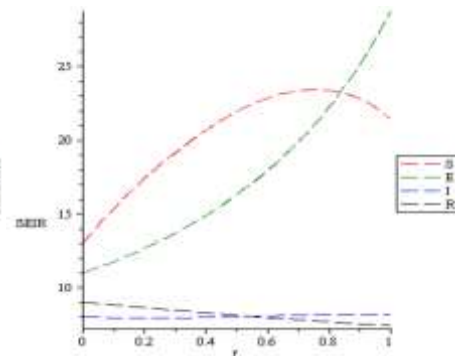
$$\begin{aligned}
R(t) &= \left. \begin{aligned} &9 - 1.9t + (0.2625000000)t^2 \\ &+ (-0.0484583333 \\ &+ 0.0693333333\beta)t^3 \\ &+ (0.0546666667\beta)t^4 \end{aligned} \right\}
\end{aligned}$$

III. Results and Discussion

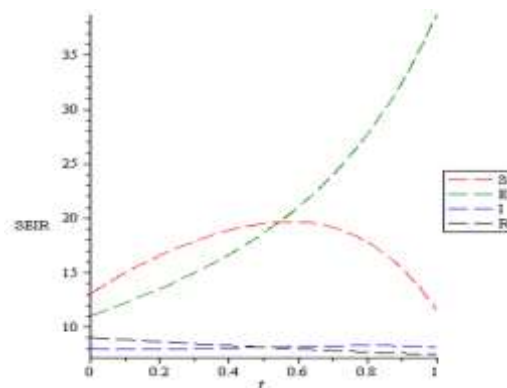
The simulation process is carried out with the aid of Maple18 software and the impact of disease transmission coefficient β is investigated in each class of the epidemic model at an interval of $0 \leq \beta \leq 1$. The results of the computer simulation are presented graphically for effective exposition

Figure 1:- Results at $\beta = 0$ Figure 2:- Results $\beta = 0.25$

$$s_0 = 13, i_0 = 8, e_0 = 11, r_0 = 9, \varepsilon = 0.25, \mu = 0.3, \gamma = 0.1, \lambda = 0.16,$$

Figure 3:- Results at $\beta = 0.5$ Figure 4:- Results at $\beta = 0.75$

$$s_0 = 13, i_0 = 8, e_0 = 11, r_0 = 9, \varepsilon = 0.25, \mu = 0.3, \gamma = 0.1, \lambda = 0.16$$

Figure 5:- Results at $\beta = 1$

$$s_0 = 13, i_0 = 8, e_0 = 11, r_0 = 9, \varepsilon = 0.25, \mu = 0.3, \gamma = 0.1, \lambda = 0.16$$

The outcome of the simulation process on the epidemic model presented in figure 1-5 reveals that the disease transmission coefficient plays a significant role in disease eradication. From figure 1, when the coefficient of the disease transmission is 0 the population of the susceptible class is at its peak as the infection rate between humans and vectors is zero. Figure 2 reveals an increase in the population of the exposed class as level of β progresses from 0 to 0.25. From figures 3 to 5 as the level of β increases from 0.5, 0.75 to 1, more people leave the susceptible class to join the exposed class. This implies that majority of the population are subjected to the risk of getting infected and if control strategy such as implementing measures that are capable of reducing the disease transmission coefficient is not taken into consideration by health workers, eradicating the disease may not be feasible. In general, it will be observed that disease transmission coefficient plays vital role in disease eradication that is the lower the value of β the better eradication.

IV. Conclusion

From the simulation results of the model, it was observed that disease transmission coefficient plays vital role in disease eradication. i.e. the lower the disease transmission coefficient, the better eradication. Also, the Laplace Adomian Decomposition Method is a powerful tool for analyzing the results because it gives a better approximation which converges to the exact solution when there exist any.

Table A: Table of Abbreviations

S/N	ABBREVIATION	MEANING
1	SEIR	Susceptible-Exposed-Infected-Recovered
2	LADM	Laplace-Adomian-Decomposition – Method
3	SEIRS	Susceptible-Exposed-Infected-Recovered-

4	HAM	Susceptible Homotopy Analysis Method
5	IVP	Initial Value Problem
6	HIV	Human Immunodeficiency Virus

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