DOI: 10.36108/ujees/5202.70.0132

UNIOSUN Journal of Engineering and Environmental Sciences. Vol. 7 No. 1. March. 2025

A Novel Mathematical Model Evaluating the Impact of Saturated Treatment Response, Vaccination and Anti-Biotic Resistance on Transmission Dynamics of Typhoid Fever

Ayoola, R. G., Kolawole, M. K. and Odeyemi, K. A.

Abstract This research presents a novel mathematical model for evaluating typhoid fever transmission, incorporating treatment response, vaccination, and antibiotic resistance. By integrating these factors, the model provides insights into disease control. We analyze the impact of saturated treatment response, vaccine efficacy, and antibiotic resistance management. A qualitative study confirms the model's epidemiological soundness through uniqueness, positivity, stability, and boundedness analyses. Sensitivity analysis, based on the reproduction number, identifies key parameters influencing disease progression. Using next-generation matrices, we establish that $(R_0 < 1)$ ensures disease-free equilibrium stability, while $(R_0 > 1)$ leads to instability. Numerical simulations via the Homotopy Perturbation Method highlight the importance of high vaccination coverage for herd immunity. Findings stress the need for integrated strategies, including vaccination, improved treatment, and responsible antibiotic use. The study concludes that treatment saturation, vaccination, and antibiotic resistance are key considerations for effective typhoid fever control.

Keywords: Typhoid Fever Transmission, Mathematical Modeling, Vaccination Efficacy, Antibiotic Resistance, Sensitivity Analysis

I. Introduction

Typhoid fever remains a major public health concern, particularly in regions with inadequate sanitation and limited healthcare access. The disease, caused by Salmonella Typhi, is primarily transmitted through contaminated food and water, leading to severe health complications if not properly managed [1]. Effective control strategies require a comprehensive understanding of the transmission dynamics, incorporating crucial factors such as treatment response, vaccination efficacy, and antibiotic resistance. Mathematical modeling has emerged as a powerful tool for studying infectious disease transmission, offering valuable insights into the effects of various intervention measures [2, 3]. We introduces a novel mathematical model that evaluates the impact of saturated treatment response, vaccination, and antibiotic resistance on the transmission dynamics of typhoid fever [4]. By integrating these critical factors, the model provides a quantitative framework to assess disease progression and control strategies. One of the primary concerns in typhoid management is antibiotic resistance, which reduces the effectiveness of treatment and complicates disease control efforts [5]. Additionally, vaccination plays a pivotal role in reducing infections, while treatment saturation where the healthcare system reaches its capacity can significantly influence disease spread [6].

Ayoola, R. G., Kolawole, M. K., Odeyemi, K. A.,
(Department of Mathematical Sciences, Osun State University, Osogbo. Nigeria)
Corresponding Author:: mutairu.kolawole@uniosun.edu.ng

This research encompasses extends to analyzing the epidemiological soundness of the proposed model through qualitative assessments, including stability analysis, positivity, uniqueness, and boundedness. Sensitivity analysis, based on the reproduction number (R_0), identifies key parameters influencing disease progression [7]. Using next-generation matrices, the study establishes the conditions for disease-free equilibrium stability and the implications of exceeding the threshold for sustained transmission [8-10]. Furthermore, numerical simulations via the Homotopy Perturbation Method highlight the significance of high vaccination coverage in achieving herd immunity by [11]. This research emphasize the need for integrated approaches combining vaccination campaigns, improved treatment accessibility, and responsible antibiotic usage [12]. By addressing these critical factors, the study aims to guide evidence-based interventions for mitigating the burden of typhoid fever and strengthening public health efforts in endemic regions [13, 14]. The critical role of vaccination, treatment response, and antibiotic resistance management in controlling typhoid fever [15]. The mathematical model provides valuable insights into disease dynamics, emphasizing the importance of high vaccination coverage and responsible antibiotic use. Sensitivity analysis identifies key parameters influencing transmission, while numerical simulations confirm the effectiveness of integrated control strategies [16, 17]. These findings reinforce the need for a comprehensive approach, combining vaccination, improved treatment, and antibiotic stewardship, to achieve sustainable disease control and prevent future outbreaks

II. Materials and Methods

A. Model Formulation

We develop a deterministic mathematical model on the transmission dynamics of Typhoid fever based on the epidemiological status of individuals in the population. The population are subdivided into different epidemiological classes: Susceptible (S), Carrier (C), Infected (I), Recovered (R) and bacteria (B) subclass. Recruitment into the susceptible population is at the rate and also from recovered class by losing temporary immunity with ω rate. The force infection of the is λ natural death rate occurs in all the classes at a rate μ, the disease induced death for infected individuals is a rate d. The parameters v, r, and δ represents the vaccination rate, antidrug resistance and saturated term respectively. The mathematical model for the system of non-linear differential equation is:

$$S' = \Lambda + \omega R - (\mu + \lambda + \nu)S$$

$$C' = \rho\lambda S - (\sigma_1 + \theta + \mu + \phi)C$$

$$I' = (1-\rho)\lambda S + \theta C - (\sigma_2 + \mu + d)I - \frac{(1-\tau)rI}{1+\delta I} \quad (1)$$

$$R' = \frac{(1-\tau)rI}{1+\delta I} + \phi C - (\mu + \omega)R + \nu S$$

$$B' = \sigma_1 C + \sigma_2 I - \mu_b B$$

Whereas $\lambda = \frac{B\nu_1}{(K+B)}$ the initial condition of $\lambda > 0$

B. Existence and uniqueness of model solution

Feasible Region; we obtained the feasible Region, in which the model solution is bounded. First we considered the total human population (N) in model solution where N = S + C + I + R + B.

Adding the above system of equation (2), we have

$$\frac{dN}{dt}=\Lambda+\omega R-(\mu+\lambda+v)S+\rho\lambda S-(\sigma_1+\theta+\mu+\phi)C+$$
$$(1-\rho)\lambda S+\theta C-(\sigma_2+\mu+d)I-\frac{(1-\tau)rI}{1+\delta I}+\frac{(1-\tau)rI}{1+\delta I}+\phi C-(\mu+\omega)R+vS$$
$$\sigma_1 C+\sigma_2 I-\mu_b B$$

$$\frac{dN}{dt}=\Lambda-\mu S-\mu C-\mu I-dI-\mu R-\sigma_1 C-\sigma_2 I \qquad (2)$$

$$\frac{dN}{dt}=\Lambda-\mu N-dI-\sigma_1 C-\sigma_2 I$$

At no outbreak of disease, C = 0 = 0, becomes

$$\frac{dN}{dt}\le\Lambda-\mu N$$

$$\frac{dN}{dt}+\mu N\le\Lambda \qquad (3)$$

$$Let\ p=\mu,\ and\ Q=\Lambda$$

By Method of integrating factor (I.F)

$$N\cdot IF=\int IF\cdot Qdt$$

$$IF=\ell^{\int pdt}$$

$$IF=\ell^{\int \mu dt}$$

$$N\cdot\ell^{\mu t}=\int\ell^{\mu t}.\Lambda dt$$

$$N\cdot\ell^{\mu t}=\Lambda\int\ell^{\mu t}$$

$$N\cdot\ell^{\mu t}=\frac{\Lambda}{\mu}\ell^{\mu t}+C$$

$$N(t)=\frac{\Lambda}{\mu}+\ell^{\mu t}C$$

$$At\quad t=0,$$

$$N(0)=\frac{\Lambda}{\mu}+C$$

$$C=N(0)-\frac{\Lambda}{\mu}$$

$$N(t)\le\left[\frac{\Lambda}{\mu}+(N(0)-\frac{\Lambda}{\mu})\cdot\ell^{-\mu t}\right]$$

$$\lim_{t\to\infty} N(t)\le\lim_{t\to\infty}\left[\frac{\Lambda}{\mu}+(N(0)-\frac{\Lambda}{\mu})\ell^{-\mu t}\right] \qquad (4)$$

$$N(t)\le\frac{\Lambda}{\mu}$$

Thus, the feasible solution of the system equation of the model enters and remains in the region. This is a positive invariant set under the flow described by (8) so that no solution path leaves through any boundary $\Re_{+}^{5}$. Hence, it is sufficient to consider the dynamics of the model in the domain $\Re_{+}^{5}$. In this region, the model can be considered has be mathematically and epidemiologically well-posed. This shows that the total population and the subpopulation $S(t), C(t), I(t), R(t), B(t)$ of the model are bounded and is a unique solution. Hence, its applicability to studying physical systems is feasible

C. Positivity and Boundedness of the Model Solution

We let the initial condition of the model to be nonnegative and now, we also proof that the solution of the model is also positive.

Theorem, let

$$\Gamma=\{(S,C,.I,R,B)\in R^5; S_0>0.C_0>0, I_0>0, R_0>0, B_0>0\};$$

Then the solution of {S,C,I, R,B} are positive for t>=0.

Proof, from the system of differential equation, we solve the equation one after the other.

First equation;

$$S'=\Lambda+\omega R-(\mu+\lambda+v)S\Rightarrow$$

$$S'(t)\ge-(\mu+\lambda+v)S(t) \qquad (5)$$

$$S'+(\mu+\lambda+v)S(t)\ge 0$$

Then solving using method of integrating factor and applying condition,
We obtained

$$S(t)\ell^{(\mu+\lambda+\nu)t} \geq \int \ell^{(\mu+\lambda+\nu)t} \cdot 0\,dt$$

$$S(t)\ell^{(\mu+\lambda+\nu)t} \geq 0 + C$$

$$S(t)\ell^{(\mu+\lambda+\nu)t} \geq C$$

$$S(t) \geq \ell^{-(\mu+\lambda+\nu)t} \cdot C$$

When t=0,
S(0)>=C

$$S(t) \geq S_0 \cdot \ell^{-(\mu+\lambda+\nu)t} \geq 0 \qquad (6)$$

Then by taking the second equation;

$$C' = \rho\lambda S - (\sigma_1 + \theta + \mu + \phi)C \Rightarrow$$

$$C'(t) \geq -(\sigma_1 + \theta + \mu + \phi)C(t)$$

$$C' + \sigma_1 + \theta + \mu + \phi)C(t) \geq 0$$

Then solving using integrating factor and applying condition, it gives

$$C(t) \cdot \ell^{(\sigma_1+\theta+\mu+\phi)t} \geq \int \ell^{(\sigma_1+\theta+\mu+\phi)t} \cdot 0\,dt$$

$$C(t) \cdot \ell^{(\sigma_1+\theta+\mu+\phi)t} \geq A + 0$$

When t=0,

$$C_0 \geq A$$

$$C(t) \geq C_0 \cdot \ell^{(\sigma_1+\theta+\mu+\phi)t} \geq 0$$

Also, we took the third equation of (3.2) which is

$$I' = (1-\rho)\lambda S + \theta C - (\sigma_2 + \mu + d)I - \frac{(1-\tau)rI}{1+\delta I}$$

We consider $\frac{1}{1+\delta I}$ and this can expanded in series form

$$(1+\delta I)^{-1} = 1 - \delta I + \delta^2 I^2 - \delta^3 I^3 + \delta^4 I^4$$

$$I' = -[\ (\sigma_2 + \mu + d)I(t)] - (1-\tau)r(1-\delta I)$$

$$I' = -\ (\sigma_2 + \mu + d) - (1 - \delta I - rI - \tau\delta I)$$

$$I' = -(\sigma_2 + \mu + d) - [\tau\delta + \delta]I$$

$$I' = -[(\sigma_2 + \mu + d) + \delta(1-\tau)I(t)] \Rightarrow \qquad (7)$$

$$\frac{I'}{I(t)} \geq -\int (\sigma_2 + \mu + d) + \delta(1-\tau)dt$$

In I(t) $\geq -[\sigma_2 + \mu + d) + \delta(1-\tau)t] + C$

I(t) $\geq \ell^{-(\sigma_2+\mu+d)+\delta(1-\tau)t} \geq 0, t \geq 0$

I(t $\geq I_0 \ell^{-(\sigma_2+\mu+d)+\delta(1-\tau)t} \geq 0$

Similarly, we took the fourth equation of which is

$$R' = \frac{(1-\tau)rI}{1+\delta I} + \phi C - (\mu + \omega)R + \nu S \Rightarrow$$

$$\frac{R'(t)}{dt} \geq -((\mu + \omega)R(t)$$

$$\frac{R'(t)}{dt} + (\mu + \omega)R(t) \geq 0$$

Then solving, using integrating factor and applying conditions

$$R(t) \cdot \ell^{(\mu+\omega)t} \geq \int \ell^{(\mu+\omega)t} \cdot 0\,dt \qquad (8)$$

$$R(t) \geq \ell^{-(\mu+\omega)t} \cdot C \geq 0$$

At t≥0

$$R(0) \geq \ell^{-(\mu+\omega)0} \cdot C \geq 0$$

$$R_0 \geq C$$

$$R(t) \geq R_0\ \ell^{-(\mu+\omega)t} \geq 0$$

Finally we took the fifth equation of

$$B'(t) = \sigma_1 C + \sigma_2 I - \mu_b B \quad \Rightarrow \qquad (9)$$

$$\frac{B'(t)}{dt} \geq -\ \mu_b B$$

$$\frac{dB(t)}{dt} + \mu_b B \geq 0$$

Then solving using method of integrating factor and applying condition which gives

$$B(t) \cdot \ell^{(\mu_b)t} \geq \int \ell^{(\mu_b)t} \cdot 0\,dt$$

$$B(t) \cdot \ell^{(\mu_b)t} \geq 0 + C$$

At t≥0

$$B(0) \geq C$$

$$B_0 \geq C$$

$$B(t) \geq B_0\, \ell^{-(\mu_b)t} \geq 0 \qquad (10)$$

This completes the proof of the theorem, therefore, the solution of the model is positive Hence the solution is bounded, therefore it is well- posed and represent a physical problem.

D. Model Disease Free Equilibrium

To find the disease free equilibrium we equate right hand side of the model to zero, evaluating it at C = I = R = B = 0 and solving for the non-infected and non-carrier state variables.

$$\Lambda + \omega R - (\mu + \lambda + \nu)S = 0$$

$$\rho\lambda S - (\sigma_1 + \theta + \mu + \phi)C = 0$$

$$(1-\rho)\lambda S + \theta C - (\sigma_2 + \mu + d)I - \frac{(1-\tau)rI}{1+\delta I} = 0$$

$$\frac{(1-\tau)rI}{1+\delta I} + \phi C - (\mu + \omega)R + \nu S = 0$$

$$\sigma_1 C + \sigma_2 I - \mu_b B = 0$$

$$\Lambda - \mu S - \lambda S - \nu S = 0 \qquad (11)$$

$$\Lambda - S(\mu + \nu) = 0$$

$$\Lambda = S(\mu + \nu)$$

$$S_0 = \frac{\Lambda}{(\mu + \nu)}$$

$$E_0 = (S, C, I, R, B) = \left(\frac{\Lambda}{(\mu + \nu)}, 0, 0, 0, 0\right)$$

E. Endemic Equilibrium Point

To obtain an endemic equilibrium $E^\bullet$, when I ≠ 0

From above we obtain $S^\bullet, C^\bullet, I^\bullet, R^\bullet, B^\bullet$;

Let $A_1 = (\mu + \lambda + \nu)$

$A_2 = (\sigma_1 + \theta + \mu + \phi)$,

$A_3 = (\sigma_2 + \mu + d)$,

$A_4 = \dfrac{(1-\tau)r}{1+\delta I}$

$$S^\bullet = \frac{A_2(-\omega^2 A_3 - \omega^2 A_4 + \Lambda A_3 + \Lambda A_4)}{-\omega A_2 V A_3 - \omega A_2 V A_4 - \omega\phi\rho\lambda A_3 - \omega\phi\rho\lambda A_4 + A_2 A_1 A_4 - \omega A_4 A_2 \lambda + \omega A_4 A_2 \rho\lambda - \omega A_4 \theta\rho\lambda}$$

$$C^\bullet = \frac{\rho\lambda(-\omega^2 A_3 - \omega^2 A_4 + \Lambda A_3 + \Lambda A_4}{-\omega A_2 V A_3 - \omega A_2 V A_4 - \omega\phi\rho\lambda A_3 + \omega\phi\rho\lambda A_4 + A_2 A_1 A_3 + A_2 A_1 A_4 - \omega A_4 A_2 \lambda + \omega A_4 A_2 \lambda + \omega A_4 A_2 \rho\lambda - \omega A_4 \theta\rho\lambda}$$

$$I^\bullet = \frac{(-\omega^2 + \Lambda)\lambda(-A_2 + A_2\rho - \theta\rho)}{-\omega A_2 V A_3 - \omega A_2 V A_4 - \omega\phi\rho\lambda A_3 - \omega\phi\rho\lambda A_4 + A_2 A_1 A_3 + A_2 A_1 A_3 + A_2 A_1 A_4 - \omega A_4 A_2 \lambda + \omega A_4 A_2 \rho\lambda}$$

$$R^\bullet = \frac{-\Lambda A_2 V A_3 - \Lambda A_2 V A_4 - \Lambda\phi\rho\lambda A_3 - \Lambda\phi\rho\lambda A_4 - \Lambda A_4 A_2 \lambda + \Lambda A_4 A_2 \rho\lambda - \Lambda A_4 \theta\rho\lambda + A_2 A_1 \omega A_3 + A_2 A_1 \omega A_4}{-\omega A_2 V A_3 - \omega A_2 V A_4 - \omega\phi\rho\lambda A_3 - \omega\phi\rho\lambda A_4 + A_2 A_1 A_3 + A_2 A_1 A_4 - \omega A_4 A_2 \lambda + \omega A_4 A_2 \rho\lambda - \omega A_4 \theta\rho\lambda}$$

$$B^\bullet = \frac{\begin{array}{l}-\lambda(\sigma_2 A_2 \omega^2 - \sigma_2 \omega^2 A_2 \rho + \sigma_2 \omega^2 \theta\rho - \sigma_2 A_2 \Lambda + \sigma_2 \Lambda A_2 \rho - \sigma_2 \Lambda\theta\rho + \sigma_1 \rho\omega^2 A_3 + \sigma_1 \rho\omega^2 A_4 \\ -\sigma_1 \rho\Lambda A_3 - \sigma_1 \rho\Lambda A_4)\end{array}}{(-\omega A_4 A_2 \rho\lambda - \omega A_2 V A_4 - \omega\phi\rho\lambda A_3 - \omega\phi\rho\lambda A_4 + A_2 A_1 A_3 + A_2 A_1 A_4 - \omega A_4 A_2 \lambda + \omega A_4 A_2 \rho\lambda - \omega A_4 \theta\rho\lambda)\mu_b}$$

F. Derivation of Basic Reproduction Number of R_0

There are two diseases state but only one way to create new infections. Hence, exposed and infected compartments of the model are involved in the calculation of R_0 from equation (3.2)

$$F_i = [\frac{\partial f_i(x_0)}{\partial x_j}] \qquad V_i = [\frac{\partial v_i(x_0)}{\partial x_j}] \quad (12)$$

F_i is the new infections, while the V_i are transfers of infections from one compartment to another.

To calculate R_0, we have;

$R_o = FV^{-1}$

$$C' = \rho\lambda S - (\sigma_1 + \theta + \mu + \phi)C$$

$$I' = (1-\rho)\lambda S + \theta C - (\sigma_2 + \mu + d)I - \frac{(1-\tau)rI}{1+\delta I}$$

$$B' = \sigma_1 C + \sigma_2 I - \mu_b B$$

Where, $\lambda = \dfrac{B\nu_1}{K+B} \qquad S_0 = \dfrac{\Lambda}{\mu+\nu}$

$$F = \begin{pmatrix} \rho\lambda S_0 \\ (1-\rho)\lambda S_0 \end{pmatrix}$$

$$F = \begin{pmatrix} \dfrac{\rho B\nu_1\Lambda}{(\nu+\mu)(K+B)} \\ \dfrac{(1-\rho)B\nu_1\Lambda}{(\nu+\mu)(K+B)} \end{pmatrix}$$

$$F = \begin{bmatrix} 0 & 0 & \dfrac{\rho\nu_1\Lambda}{(\nu+\mu)K} \\ 0 & 0 & \dfrac{(1-\rho)\nu_1\Lambda}{(\nu+\mu)K} \\ 0 & 0 & 0 \end{bmatrix}$$

$$V = \begin{pmatrix} (\sigma_1 + \theta + \mu + \phi)C \\ (1-\rho)\lambda S + \theta C - (\sigma_2 + \mu + d)I - \dfrac{(1-\tau)rI}{1+\delta I} \\ \sigma_1 C + \sigma_2 I - \mu_b B \end{pmatrix}$$

$$V = \begin{bmatrix} (\sigma_1+\theta+\mu+\phi) & 0 & 0 \\ -\theta & +(\sigma_2+\mu+d) - \dfrac{(1-\tau)r}{1+\delta} & 0 \\ -\sigma_1 & -\sigma_2 & \mu_b \end{bmatrix} \quad (13)$$

Now let

$K_1 = \sigma_1 + \theta + \mu + \phi),$

$K_2 = (\sigma_2 + \mu + d) - \dfrac{(1-\tau)r}{1+\delta},$

$K_3 = \mu_b, A = -\theta, B = -\sigma_1, c = -\sigma_2 \quad (3.32)$

$|V| = K_1 K_2 K_3$

$$V^{-1} = \frac{V_c^{\,T}}{|V|} \qquad V = \begin{bmatrix} K_1 & 0 & 0 \\ A & K_2 & 0 \\ B & c & K_3 \end{bmatrix}$$

$$V_c = \begin{bmatrix} K_1K_3 & AK_3 & Ac - BK_2 \\ 0 & K_1K_3 & K_1c \\ 0 & 0 & K_1K_2 \end{bmatrix}$$

$$V_c^{\,T} = \begin{bmatrix} K_2K_3 & 0 & 0 \\ AK_3 & K_1K_3 & 0 \\ Ac - BK_2 & K_1c & K_1K_2 \end{bmatrix}$$

$$V^{-1} = \frac{V_c^{\,T}}{|V|}$$

$$= \frac{1}{K_1K_2K_3} \begin{bmatrix} K_2K_3 & 0 & 0 \\ AK_3 & K_1K_3 & 0 \\ Ac - BK_2 & K_1c & K_1K_2 \end{bmatrix} \quad (14)$$

$$V^{-1} = \begin{pmatrix} K_1 & 0 & 0 \\ \dfrac{A}{K_1K_2} & \dfrac{1}{K_2} & 0 \\ \dfrac{Ac - BK_2}{K_1K_2K_3} & \dfrac{c}{K_2K_3} & \dfrac{1}{K_3} \end{pmatrix}$$

Where

$R_0 = FV^{-1}$

$$FV^{-1}=\begin{pmatrix} 0 & 0 & \frac{\rho v_1\Lambda}{(v+\mu)K} \\ 0 & 0 & \frac{(1-\rho)V_1\Lambda}{(v+\mu)K} \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & \frac{K_1\rho v_1\Lambda}{(v+\mu)K} \\ 0 & 0 & \frac{A\rho v_1\Lambda}{(v+\mu)KK_1K_2}+\frac{c(1-\rho)v_1\Lambda}{(v+\mu)K_2K} \\ 0 & 0 & \frac{(Ac-BK_2)\rho v_1\Lambda}{K_1K_2K_3(v+\mu)K}+\frac{c(1-\rho)v_1\Lambda}{(v+\mu)K_2K_3K_1K} \end{pmatrix}$$

The most positive Eigen value is our R_0, therefore

$$R_0=\frac{v_1\Lambda\left[(Ac-BK_2)\rho+K_1(1-\rho)c\right]}{K_1K_2K_3(v+\mu)K} \quad (15)$$

Now putting (3.32) into (3.34), gives

$$R_0=\frac{\Lambda\ v_1\left[(\theta\sigma_2+\sigma_1(\sigma_2+\mu+d)+\frac{(1-\tau)r)}{\delta})\rho+(\tau_1+\theta+\mu+\phi)(1-\rho)-\sigma_2\right]}{K_1K_2K_3(v+\mu)K}$$

$$=\frac{\Lambda v_1\left[\delta(\theta\sigma_2+\sigma_1(\sigma_2+\mu+d)+(1-\tau)r)\rho+\delta(\tau_1+\theta+\mu+\phi)(1-\rho)-\sigma_2\right]}{\delta K_1K_2K_3(v+\mu)K}$$

$$R_0=\Lambda v_1\left[\frac{(\theta\sigma_2+\sigma_1K_2)\rho-K_1(1-\rho)\sigma_2}{K_1K_2K_3(V+\mu)K}\right] \quad (16)$$

G. Local Stability of Disease-free Equilibrium

The disease free equilibrium of the proposed epidemic model is locally asymptotically stable if $R_0<1$ and unstable whenever $R_0>1$

The local stability of the disease free equilibrium at $S_0=\frac{\Lambda}{(\mu+v)}$ as

$$E_1=\left(\frac{\Lambda}{(\mu+v)},0,0,0,0\right)$$

The Jacobian matrix of the system of (3.2) was considered and differentiate each compartment with respect to (S, C, I, R, B) and applying Lassalle's principle.

where $|J_E-\lambda_i I|=0$ *as* λ_i *and* I Are the Eigen –values and identity matrix respectively. Where i= 1, 2, 3, 4.

Therefore:

$$J_{E_0}=\begin{vmatrix} -(\mu+\lambda+v) & 0 & 0 & \omega & 0 \\ \rho\lambda & -(\sigma_1+\theta+\mu+\phi) & 0 & 0 & 0 \\ (1-\rho)\lambda & \theta & -(\sigma_2+\mu+d)-\frac{(1-\tau)r}{\delta} & 0 & 0 \\ v & \phi & \frac{(1-\tau)r}{\delta} & -(\mu+\omega) & 0 \\ 0 & \sigma_1 & \sigma_2 & 0 & -\mu_b \end{vmatrix} \quad (17)$$

At Equilibrium, I=C=R=B=0, $S_0=\frac{\Lambda}{(\mu+v)}$

Now equation (3.38) becomes

$$J_{E_0}=\begin{vmatrix} -(\mu+v)-\lambda_1 & 0 & 0 & \omega & 0 \\ 0 & -(\sigma_1+\theta+\mu+\phi)-\lambda_2 & 0 & 0 & 0 \\ 0 & \theta & -(\sigma_2+\mu+d)-\frac{(1-\tau)r}{\delta}-\lambda_3 & 0 & 0 \\ v & \phi & \frac{(1-\tau)r}{\delta} & -(\mu+\omega)-\lambda_4 & 0 \\ 0 & \sigma_1 & \sigma_2 & 0 & -\mu_b-\lambda_5 \end{vmatrix}=0$$

$$\left|J_{E_0}-\lambda I\right|=0$$

Using Atangana Belame invariance principle by lower triangular matrix,

We obtain

$$\begin{vmatrix} -(\mu+\omega)-\lambda_4 & 0 \\ 0 & -\mu_b-\lambda_5 \end{vmatrix}=0 \quad (18)$$

$$\left[-(\mu+\omega)-\lambda_4\right]\left[-\mu_b-\lambda\right]=0$$

$$\Rightarrow \qquad \lambda_4=-(\mu+\varpi),\quad \lambda_5=-\mu_b<0,$$

$$\begin{vmatrix} -(\sigma_2+\mu+d)-\frac{(1-\tau)r}{\delta}-\lambda_3 & 0 & 0 \\ \frac{(1-\tau)r}{\delta} & -(\mu+\omega)-\lambda_4 & 0 \\ \sigma_2 & 0 & -\mu_b-\lambda_5 \end{vmatrix}=0$$

Then from () we obtained

$$(-(\sigma_2+\mu+d)-\frac{(1-\tau)r}{\delta}-\lambda_3)\begin{vmatrix}-(\mu+\omega)-\lambda_4 & 0\\ 0 & -\mu_b-\lambda_5\end{vmatrix}=0$$

$$(-(\sigma_2+\mu+d)-\frac{(1-\tau)r}{\delta}-\lambda_3)(-(\mu+\omega)-\lambda)(\mu_b-\lambda)=0$$

$$\Rightarrow \lambda_3=-(\sigma_2+\mu+d)-\frac{(1-\tau)r}{\delta}<0 \qquad (19)$$

$$\begin{vmatrix}-(\mu+v)-\lambda_1 & 0 & 0 & \omega & 0\\ 0 & -(\sigma_1+\theta+\mu+\phi)-\lambda_2 & 0 & 0 & 0\\ 0 & \theta & -(\sigma_2+\mu+d)-\frac{(1-\tau)r}{\delta}-\lambda_3 & 0 & 0\\ v & \phi & \frac{(1-\tau)r}{\delta} & -(\mu+\omega)-\lambda_4 & 0\\ 0 & \sigma_1 & \sigma_2 & 0 & -\mu_b-\lambda_5\end{vmatrix}=0$$

From (19), we obtain

$$-(\sigma_1+\theta+\mu+\phi)-\lambda_2\times\begin{vmatrix}-[\frac{\delta(\sigma_2+\mu+d)+(1-\tau)r}{\delta}]-\lambda_3 & 0 & 0\\ \frac{(1-\tau)r}{\delta} & -(\mu+\omega)-\lambda_4 & 0\\ \sigma_2 & 0 & -\mu_b-\lambda_5\end{vmatrix}=$$

Then this results to,

$$(-(\sigma_1+\theta+\mu+\phi)-\lambda_2)((-[\frac{\delta(\sigma_2+\mu+d)+(1-\tau)r}{\delta}]-\lambda_3)\times\begin{vmatrix}-(\mu+\omega)-\lambda_4 & 0\\ 0 & -\mu_b-\lambda_5\end{vmatrix}=0$$

$$(-(\sigma_1+\theta+\mu+\phi)-\lambda_2)(-(\sigma_2+\mu+d)(-\frac{(1-\tau)r}{\delta})-\lambda_3)(-(\mu+\omega)-\lambda_4)(\mu_b-\lambda_5)=0$$

$$\Rightarrow \lambda_2=-(\sigma_1+\theta+\mu+\phi)$$

Also from (19), we obtained

$$\lambda_1=-(\mu+v),\ \lambda_2=-(\sigma_1+\theta+\mu+\phi),\lambda_3=-[\frac{\delta(\sigma_2+\mu+d)+(1-\tau)r}{\delta}],\lambda_4=-(\mu+\omega),\quad (20)$$
$$\lambda_5=-\mu_b$$

Since all the eigen values are all negative, hence the disease free equilibrium is locally asymptotically stable.

H. Local stability for endemic

Theorem: The Endemic Equilibrium of the proposed epidemic model is locally asymptotically stable if $R_0<1$ and unstable otherwise.

Proof:

We linearized each of the compartment in

Let

$$S=x+S^*,C=y+C^*,I=z+I^*,R=a+R^*,B=b+B^*$$

$$S'=\Lambda+\omega(a+R^*)-(\mu+\lambda+)(x+S^*)$$
$$C'=\rho\lambda(x+S^*)-(\sigma_1+\theta+\mu+\phi)(z+I^*)$$
$$I'=(1-\rho)\lambda(x+S^*)+\theta(y+C^*)-(\sigma_2+\mu+d)-\frac{(1-\tau)r(z+I^*)}{1+\delta(z+I^*)}$$
$$R'=\frac{(1-\tau)r(z+I^*)}{1+\delta(z+I^*)}+\phi(y+C^*)-(\mu+\omega)(a+R^*)+v(x+S^*)$$

Then we substitute to the above equation (20)

$$\frac{dx}{dt}=\Lambda+\omega a-(\mu+\lambda+v)x$$
$$\frac{dy}{dt}=\rho\lambda x-(\sigma_1+\theta+\mu+\phi)z$$
$$\frac{dz}{dt}=(1-\rho)\lambda x+\theta y-(\sigma_2+\mu+d)z-(1-\tau)rz(\delta z)^{-1} \qquad (21)$$
$$\frac{da}{dt}=(1-\tau)rz(\delta z)^{-1}+\phi y-(\mu+\omega)a+vx$$
$$\frac{db}{dt}=\sigma_1 y+\sigma_2 z-\mu_b b$$

Then we differentiate each compartment one by one and take the Jacobian- matrix

$$J_{E_E}=\begin{vmatrix}-(\mu+\lambda+v) & 0 & 0 & \omega & 0\\ \rho\lambda & 0 & -(\sigma_1+\theta+\mu+\phi) & 0 & 0\\ (1-\rho)\lambda & \theta & \left[(\sigma_2+\mu+d)+\frac{(1-\tau)r}{\delta}\right] & 0 & 0\\ v & \phi & \frac{(1-\tau)r}{\delta} & -(\mu+\omega) & 0\\ 0 & \sigma_1 & \sigma_2 & 0 & -\mu_b\end{vmatrix}$$

$$\left|J_{E_E}-\lambda I\right|=0$$

$$J_{E_E} = \begin{Vmatrix} -(\mu+\lambda+v)-\lambda_* & 0 & 0 & \omega & 0 \\ \rho\lambda & 0-\lambda_* & -(\sigma_1+\theta+\mu+\phi) & 0 & 0 \\ (1-\rho)\lambda & \theta & \left[(\sigma_2+\mu+d)+\frac{(1-\tau)r}{\delta}\right]-\lambda_* & 0 & 0 \\ v & \phi & \frac{(1-\tau)r}{\delta} & -(\mu+\omega)-\lambda_* & 0 \\ 0 & \sigma_1 & \sigma_2 & 0 & -\mu_b-\lambda_* \end{Vmatrix} = 0$$

Let A= $-(\mu+\lambda+v)$, B_1=0, C= $-[(\sigma_2+\mu+d)+\frac{(1-\tau)r}{\delta}]$, D= $-(\mu+\omega)$, E=- μ_b

From we obtain

$$(A-\lambda_*)(B_1-\lambda_*)(C-\lambda_*)(D-\lambda_*)(E-\lambda_*)=0$$

$$(AB_1-\lambda_*A-\lambda_*B_1+\lambda_*^2)(CD-\lambda_*C-\lambda_*D+\lambda_*^2)(E-\lambda_*)=0$$

$$(AB_1CD-\lambda_*AB_1C-\lambda_*AB_1D+\lambda_*^2AB_1-\lambda_*ACD+\lambda_*^2AC+\lambda_*^2AD-\lambda_*^3A-\lambda_*B_1CD$$
$$+\lambda_*^2B_1C+\lambda_*^2B_1D-\lambda_*^3B_1+\lambda_*^2CD-\lambda_*^3D+\lambda_*^4)(E-\lambda_*)=0$$

$$(AB_1CDE-\lambda_*AB_1CE-\lambda_*AB_1DE+\lambda_*^2AB_1E-\lambda_*ACDE+\lambda_*^2ACE+\lambda_*^2ADE-\lambda_*^3AE$$
$$-\lambda_*B_1CDE+\lambda_*^2B_1CE+\lambda_*^2B_1DE-\lambda_*^3B_1E+\lambda_*^2CDE-\lambda_*^3CE-\lambda_*^3DE+\lambda_*^4E-\lambda_*AB_1CD$$
$$+\lambda_*^2AB_1C+\lambda_*^2AB_1D-\lambda_*^3AB_1+\lambda_*^2ACD-\lambda_*^3AD+\lambda_*^4A+\lambda_*^2B_1CD-\lambda_*^3B_1C-\lambda_*^3B_1D+\lambda_*^4B_1$$
$$-\lambda_*^3CD+\lambda_*^4A+\lambda_*^2B_1CD-\lambda_*^3B_1C-\lambda_*^3B_1D+\lambda_*^4B_1-\lambda_*^3CD+\lambda_*^4C+\lambda_*^4D-\lambda_*^5=0$$

$$-\lambda_*^5+\lambda_*^4(A+B_1+C+D+E)-\lambda_*^3(AE+B_1E+CE+DE+AD+B_1D+CD+B_1C+AB_1)$$
$$+\lambda_*^2(AB_1E+ACE+ADE+B_1CE+B_1DE+CDE+AB_1C+AB_1D+ACD+B_1CD)$$
$$-\lambda_*(AB_1CE+AB_1DE+ACDE+B_1CDE+AB_1CE)+AB_1CDE=0$$

$$-\lambda_*^5+\lambda_*^4\{-(\mu+\lambda+v)+0-[(\sigma_2+\mu+d)+\frac{(1-\tau)r}{\delta}]-(\mu+\omega)-\mu_b\}$$
$$-\lambda_*^3\{-(\mu+\lambda+v)\times-\mu_b+0+[(\sigma_2+\mu+d)+\frac{(1-\tau)r}{\delta}]\times\mu_b)+(\mu+\omega)\times\mu_b$$
$$+(\mu+\lambda+v)\times(\mu+\omega)+0+[(\sigma_2+\mu+d)+\frac{(1-\tau)r}{\delta}])\times(\mu+\omega)\}+\lambda_*^2\{0+(-(\mu+\lambda+v)$$
$$-[(\sigma_2+\mu+d)+\frac{(1-\tau)r}{\delta}]\times-\mu_b)]+[-(\mu+\lambda+v)\times-(\mu+\omega)\times-\mu_b]+0+0+[(\sigma_2+\mu+d)$$
$$+\frac{(1-\tau)r}{\delta}]\times-(\mu+\omega)\times-\mu_b+0+0+(-(\mu+\lambda+v)\times-[(\sigma_2+\mu+d)+\frac{(1-\tau)r}{\delta}]\times-(\mu+\omega)+0\}-\lambda_*$$
$$\{0+0+-(\mu+\lambda+v)+0-(\mu+\lambda+v)\times-[(\sigma_2+\mu+d)+\frac{(1-\tau)r}{\delta}]\times-(\mu+\omega)\times-\mu_b\}+0=0$$

The characteristics polynomial J is given by

$$\lambda_*^5+a_1\lambda_*^4+a_2\lambda_*^3+a_3\lambda_*^2+a_4\lambda_*^1+a_5$$

Comparing equation (3) with equation

$$a_0=1$$

$$a_1=-\{(\mu+\lambda+v)+[(\sigma_2+\mu+d)+\frac{(1-\tau)r}{\delta}]+(\mu+\omega)+\mu_b\}$$

$$a_2=-\{(\mu+\lambda+v)\mu_b+([(\sigma_2+\mu+d)+\frac{(1-\tau)r}{\delta}]\mu_b)+((\mu+\omega)\mu_b)+(\mu+\lambda+v)(\mu+\omega)$$
$$+([\sigma_2+\mu+d)+\frac{(1-\tau)r}{\delta}])(\mu+\omega)\}$$

$$a_3=-\{(\mu+\lambda+v)([(\sigma_2+\mu+d)+\frac{(1-\tau)r}{\delta}])\mu_b)+$$
$$[(\mu+\lambda+v)(\mu+\omega)\mu_b)+([(\sigma_2+\mu+d)+\frac{(1-\tau)r}{\delta}](\mu+\omega)(\mu_b)+ \tag{22}$$
$$(\mu+\lambda+v)([(\sigma_2+\mu+d)+\frac{(1-\tau)r}{\delta}])(\mu+\omega)\}$$

$$a_4=-\{(\mu+\lambda+v)+(\mu+\lambda+v)([(\sigma_2+\mu+d)+\frac{(1-\tau)r}{\delta}])(\mu+\omega)(\mu_b)\}$$

$$a_5=0$$

Using the Routh-Hurwitz criterion, it can be seen that all the eigen values of the characteristics equation above have negative real part.

Then the endemic equilibrium is locally asymptotically stable.

I. Global Stability At Disease Free Equilibrium

To investigate the global stability, consider the Lyapunov function. Then From equation we analyse

$$C^{'}=\rho\lambda S-(\sigma_1+\theta+\mu+\phi)C,$$

$$I^{'}=(1-\rho)\lambda S+\theta C-(\sigma_2+\mu+d)I-\frac{(1-\tau)rI}{1+\delta I}$$

$$B^{'}=\sigma_1 C+\sigma_2 I-\mu_b B$$

Let $C=I_1, I=I_2, B=I_3$

Consider V (S, C, I, R, B, t) = $C_1I_1+C_2I_2+C_3I_3$.

$$\frac{dV_\bullet}{dt}=C_1I_1^{'}+C_2I_2^{'}+C_3I_3^{'}$$

Assume that $C_1 < C_2 < C_3 \Rightarrow C_3 > C_2 > C_1 \Rightarrow C_3 \geq 0$ Which gives

$$\left.\begin{array}{l} \leq [C_2\theta + C_3\sigma_1 - C_1(\sigma_1+\theta+\mu+\phi)]I_1 - C_2(\sigma_2+\mu+d)I_2 \\ - C_2[(1-\tau)rI_2(1+\delta)^{-1}]I_2 + C_1\rho\lambda S_0 + C_2(1-\rho)\lambda S_0 \end{array}\right\}$$

$$S_0 = \frac{\Lambda}{(\mu+v)}, C_1 = \frac{1}{(\sigma_1+\theta+\mu+\phi)}, C_2 = \frac{(\sigma_1+\theta+\mu+\phi)(1+\delta)r(1-\tau)}{(\sigma_2+\mu+d)}, C_3 = 0 \quad (23)$$

Then put (3) into (3), we obtain

$$\left.\begin{array}{l} \leq \left[\frac{\theta(\sigma_1+\theta+\mu+\phi)(1+\delta)r(1-\tau)}{(\sigma_2+\mu+d)(\mu+v)} - \frac{(\sigma_1+\theta+\mu+\phi)}{(\sigma_1+\theta+\mu+\phi)}\right]I_1 - \\ \left[\frac{(\sigma_2+\mu+d)(1+\delta)r(1-\tau)}{(\sigma_2+\mu+d)(\mu+v)} + \frac{(1-\tau)}{(\mu+v)} + \frac{(1-\tau)r(1+\delta)}{(1-\tau)r(1+\delta)} + \frac{\Lambda v}{(\mu+v)} \frac{\sigma_1+\theta+\mu+\phi)(1+\delta)r(1-\tau)}{(\sigma_2+\mu+d)}\right]I_2 \end{array}\right\}$$

$$\Rightarrow \leq \left[\frac{\theta(\sigma_1+\theta+\mu+\phi)(1+\delta)r(1-\tau)}{(\sigma_2+\mu+d)(\mu+v)} - 1\right]I_1 + \left[\frac{(\sigma_1+\theta+\mu+\phi)(1+\delta)r(1-\tau)}{(\sigma_2+\mu+d)} + \frac{\Lambda v}{(\mu+v)}\right]I_2 \Big\}$$

$$\leq \Gamma[R_0 - 1]I,$$

By LaSalle's principle, at equilibrium, $\Gamma = \frac{v_1\Lambda}{(\mu+v)}$, as t→∞, as $S_o = \frac{\Lambda}{(\mu+v)}$ the global stability of the disease free equilibrium at as t→∞ it is stand that whenever $R_0 < 1$, is Globally Asymptotically stable.

J. Global Stability for Endemic Equilibrium

Theorem: The model is said to have no periodic orbit

Proof: Employing the Dulac's criterion as adopted in 2021 by Ahmad et al.

Define M = (S, C, I, R, B). Be defined as the Dulac's function as; G= 1/SC

Let GM be computed as follows;

$$GS' = \frac{1}{SC}[\Lambda + \omega R - (\mu+\lambda+v)S]$$

$$GC' = \frac{1}{SC}[\rho\lambda S - (\sigma_1+\theta+\mu+\phi)C]$$

$$GI' = \frac{1}{SC}\left[(1-\rho)\lambda S + \theta C - (\sigma_2+\mu+d+\frac{(1-\tau)r)I}{1+\delta I}\right] \quad (24)$$

$$GR' = \frac{1}{SC}\left[\frac{(1-\tau)rI}{1+\delta I} + \phi C - (\mu+\omega)R + vS\right]$$

$$GB' = \frac{1}{SC}[\sigma_1 C + \sigma_2 I - \mu_b B]$$

Then $\frac{d}{dt}(GM)$ is obtained as follow;

$$\frac{d}{dt}(GM) = \frac{\partial}{\partial S}\left\{G(\frac{dS}{dt})\right\} + \frac{\partial}{\partial C}\left\{G(\frac{dC}{dt})\right\} + \frac{\partial}{\partial I}\left\{G(\frac{dI}{dt})\right\} + \frac{\partial}{\partial R}\left\{G(\frac{dR}{dt})\right\} + \frac{\partial}{\partial B}\left\{G(\frac{dB}{dt})\right\}$$

$$\frac{d}{dt}(GM) = \frac{\partial}{\partial S}\left\{\frac{\Lambda}{SC} + \frac{\omega R}{SC} - \frac{(\mu+\mu+v)}{C}\right\} + \frac{\partial}{\partial C}\left\{\frac{\rho\lambda}{C} - \frac{(\sigma_1+\theta+\mu+\phi)}{S}\right\} +$$
$$\frac{\partial}{\partial I}\left\{\frac{(1-\rho)\lambda}{C} + \frac{\theta}{S} - \frac{(\sigma_2+\mu+d+(1-\tau)r)I}{SC(1+\delta I)}\right\} + \frac{\partial}{\partial R}\left\{\frac{(1-\tau)rI}{SC(1+\delta I)} + \frac{\phi}{S} - \frac{(\mu+\omega)R}{SC} + \frac{v}{C}\right\} +$$
$$\frac{\partial}{\partial B}\left\{\frac{\sigma_1}{S} + \frac{\sigma_2 I}{SC} - \frac{\mu_b B}{SC}\right\}$$

Now, we consider the parameter with and without state variables i.e those without are negative invariant as those with states variables are neglected not relevance to SC.

$$\frac{d}{dt}(GM) = \left\{-\frac{\Lambda}{SC} - \frac{(\mu+\lambda+v)}{C}\right\} + \left\{-\frac{\rho\lambda}{C} - \frac{(\sigma_1+\theta+\mu+\phi)}{SC}\right\} + \left\{-\frac{(1-\rho)\lambda}{SC} - \frac{\theta}{C}\right\} +$$
$$\left\{-\frac{(\mu+\omega)+v}{C}\right\} + \left\{-\frac{\sigma_2}{SC} - \frac{\mu_b}{C}\right\}$$

$$= -\left[\begin{array}{l} \frac{\Lambda}{SC} + \frac{(\mu+\lambda+v}{C} + \frac{\rho\lambda}{C} + \frac{(\sigma_1+\theta+\mu+\phi)}{SC} + \frac{(1-\rho)\lambda}{SC} + \frac{\theta}{C} + \frac{(\mu+\omega)}{C} + \\ \frac{v}{C} + \frac{\sigma_2}{SC} + \frac{\mu_b}{C} \end{array}\right] \quad (25)$$

$$= -\left[\frac{\Lambda}{SC} + \frac{[(\mu+\lambda+v) + \rho\lambda + (\sigma_1+\theta+\mu+\phi) + \lambda - \rho\lambda + \theta + \mu + \omega + \sigma_2 + \mu_b]}{SC}\right]$$

$$= -\left[\frac{\Lambda}{SC} + \frac{(3\mu+\lambda+v+2\theta+\sigma_1+\omega+\phi+\sigma_2+\mu_b)}{SC}\right] < 0$$

It implies that the system has no closed orbits. Epidemically the non-existence of periodic orbits implies that there are fluctuations in the number of infective which makes it difficult to allocate resources for the control of the disease.

K. Sensitivity Analysis of R_0

We are to test for the sensitivity of R_0 by differentiating R_0 with respect to all the parameters in R_0. The normalized forward sensitivity index is defined as $\zeta_p = \frac{\partial R_0}{\partial p} \times \frac{p}{R_0}$ as

$$R_0 = \frac{\left[\Lambda v_1(\theta\sigma_2 + \sigma_1 K_2)\rho - K_1(1-\rho)\sigma_2\right]}{K_1 K_2 K_3(\mu + v)K}$$

Where $K_1 = (\sigma_1 + \theta + \mu + \phi)$, $K_2 = (\sigma_2 + \mu + d) + \frac{(1-\tau)r}{\delta}$, $K_3 = \mu_b$

$$\frac{\partial R_0}{\partial \Lambda} = \frac{\partial R_0}{\partial \Lambda} \times \frac{\Lambda}{R_0}$$

$$\frac{\partial R_0}{\partial \Lambda} = \frac{v_1[\theta\sigma_2 + \sigma_1 K_2)\rho - K_1(1-\rho)\sigma_2]}{K_1K_2K_3(\mu+v)K} \times \frac{\Lambda K_1K_2K_3(\mu+v)K}{\Lambda v_1(\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2} = 1 \quad (26)$$

$$\frac{\partial R_0}{\partial v_1} = \frac{\partial R_0}{\partial v_1} \times \frac{v_1}{R_0}$$

$$\frac{\partial R_0}{\partial v_1} = \frac{\Lambda[\theta\sigma_2 + \sigma_1 K_2)\rho - K_1(1-\rho)\sigma_2]}{K_1K_2K_3(\mu+v)K} \times \frac{V_1K_1K_2K_3(\mu+v)K}{\Lambda v_1(\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2} = 1$$

$$\frac{\partial R_0}{\partial \theta} = \frac{\partial R_0}{\partial \theta} \times \frac{\theta}{R_0}$$

$$\frac{\partial R_0}{\partial \theta} = \frac{\Lambda v_1[(\sigma_2 + \sigma_1K_2)\rho - (\sigma_1 + \mu + \phi)(1-\rho)\sigma_2]}{K_1K_2K_3(\sigma_1+\mu+\phi)(\mu+v)K} \times \frac{\theta K_1K_2K_3(\mu+v)K}{\Lambda v_1(\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2} =$$

$$\frac{[\sigma_2 + \sigma_1K_2)\rho - (\sigma_1 + \mu + \phi)(1-\rho)\sigma_2]}{(\sigma_1+\mu+\phi)} \times \frac{\theta K_1}{(\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2}$$

$$\frac{\partial R_0}{\partial \theta} = \frac{[(\sigma_2 + \sigma_1K_2)\rho - (\sigma_1 + \mu + \phi)(1-\rho)\sigma_2]\theta K_1}{(\sigma_1+\mu+\phi)(\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2} \quad (3.80)$$

$$\frac{\partial R_0}{\partial \sigma_1} = \frac{\partial R_0}{\partial \sigma_1} \times \frac{\sigma_1}{R_0}$$

$$\frac{\partial R_0}{\partial \sigma_1} = \frac{\Lambda v_1[(\theta\sigma_2 + K_2)\rho - (\theta + \mu + \phi)(1-\rho)\sigma_2]}{(\theta+\mu+\phi)K_2K_3(\mu+v)K} \times \frac{\sigma_1K_1K_2K_3(\mu+v)K}{\Lambda v_1(\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2}$$

$$= \frac{[(\theta\sigma_2 + K_2)\rho - (\theta + \mu + \phi)(1-\rho)\sigma_2]\sigma_1K_1}{(\theta+\mu+\phi)(\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2}$$

$$\frac{\partial R_0}{\partial \sigma_2} = \frac{\partial R_0}{\partial \sigma_2} \times \frac{\sigma_2}{R_0}$$

$$\frac{\partial R_0}{\partial \sigma_2} = \frac{\Lambda v_1\left\{[(\theta + \sigma_1(\mu+d) + \frac{(1-\tau)r)}{\delta}]\rho - K_1(1-\rho)\right\}}{K_1(\mu+d) + \frac{(1-\tau)r}{\delta}K_3(v+\mu)K} \times \frac{\sigma_2K_1K_2K_3(v+\mu)K}{\Lambda v_1(\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2}$$

$$= \frac{\left\{[(\theta + \sigma_1(\mu+d) + \frac{(1-\tau)r)}{\delta}]\rho - K_1(1-\rho)\right\}\sigma_2K_2}{\{(\mu+d) + \frac{(1-\tau)r}{\delta}\}(\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2}$$

$$\frac{\partial R_0}{\partial d} = \frac{\partial R_0}{\partial d} \times \frac{d}{R_0}$$

$$\frac{\partial R_0}{\partial d} = \frac{\Lambda v_1\left\{\theta + \sigma_1(\mu+\sigma_2) + \frac{(1-\tau)r)\rho}{\delta} - K_1(1-\rho)\right\}}{(K_1(\mu+\sigma_2) + \frac{(1-\tau)r}{\delta})K_3(v+\mu)K} \times \frac{dK_1K_2K_3(v+\mu)K}{\Lambda v_1(\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2}$$

$$= \frac{\left\{\theta + \sigma_1(\mu+\sigma_2) + \frac{(1-\tau)r)\rho}{\delta} - K_1(1-\rho)\right\}dK_2}{\{(\mu+\sigma_2) + \frac{(1-\tau)r}{\delta}\}(\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2}$$

$$\frac{\partial R_0}{\partial \rho} = \frac{\partial R_0}{\partial \rho} \times \frac{\rho}{R_0}$$

$$\frac{\partial R_0}{\partial \rho} = \frac{\Lambda v_1[(\theta\sigma_2 + \sigma_1K_2) + K_1\sigma_2]}{K_1K_2K_3(\mu+v)K} \times \frac{\rho K_1K_2K_3(\mu+v)K}{\Lambda v_1(\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2}$$

$$= \frac{[(\theta\sigma_2 + \sigma_1K_2) + K_1\sigma_2]\rho}{(\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2}$$

$$\frac{\partial R_0}{\partial r} = \frac{\partial R_0}{\partial r} \times \frac{r}{R_0}$$

$$= \frac{\left[[(\theta\sigma_2 + \sigma_1(\sigma_2+\mu+d) + \frac{(1-\tau)}{\delta})]\rho - K_1(1-\rho)\sigma_2\right]rK_2}{[(\sigma_2+\mu+d) + \frac{(1-\tau)}{\delta}](\theta\sigma_2 + \sigma_1K_2)]\rho - K_1(1-\rho)\sigma_2} \quad (27)$$

$$\frac{\partial R_0}{\partial V} = \frac{\partial R_0}{\partial V} \times \frac{V}{R_0}$$

$$\frac{\partial R_0}{\partial V} = \frac{\Lambda v_1[\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2]}{K_1K_2K_3\mu K} \times \frac{vK_1K_2K_3(\mu+v)K}{\Lambda v_1(\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2}$$

$$= \frac{(\mu+v)v}{\mu}$$

$$\frac{\partial R_0}{\partial K} = \frac{\partial R_0}{\partial K} \times \frac{K}{R_0}$$

$$\frac{\partial R_0}{\partial K} = \frac{\Lambda v_1[\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2]}{K_1K_2K_3(\mu+v)} \times \frac{KK_1K_2K_3(\mu+v)K}{\Lambda v_1(\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2}$$

$$= K^2$$

$$\frac{\partial R_0}{\partial \mu} = \frac{\partial R_0}{\partial \mu} \times \frac{\mu}{R_0}$$

$$\frac{\partial R_0}{\partial \mu} = \frac{\Lambda v_1\left[[(\theta\sigma_2 + \sigma_1(\sigma_2+d) + \frac{(1-\tau)r}{\delta})]\rho - K_1(1-\rho)\sigma_2\right]}{K_1[(\sigma_2+d) + \frac{(1-\tau)r}{\delta}]K_3vK} \times \frac{\mu K_1K_2K_3(\mu+v)K}{\Lambda v_1(\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2}$$

$$= \frac{\left[[(\theta\sigma_2 + \sigma_1(\sigma_2+d) + \frac{(1-\tau)r}{\delta})]\rho - K_1(1-\rho)\sigma_2\right]\mu K_2(\mu+v)}{[(\sigma_2+d) + \frac{(1-\tau)r}{\delta}](\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2 v} \quad (28)$$

$$\frac{\partial R_0}{\partial \delta} = \frac{\partial R_0}{\partial \delta} \times \frac{\delta}{R_0}$$

$$\frac{\partial R_0}{\partial \delta} = \frac{\Lambda v_1[[(\theta\sigma_2 + \sigma_1(\sigma_2+\mu+d) + (1-\tau)]\rho - K_1(1-\rho)\sigma_2]}{K_1[(\sigma_2+\mu+d) + (1-\tau)]K_3(\mu+v)K} \times \frac{\delta K_1K_2K_3(\mu+v)K}{\Lambda v_1(\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2}$$

$$= \frac{[[(\theta\sigma_2 + \sigma_1(\sigma_2+\mu+d) + (1-\tau)]\rho - K_1(1-\rho)\sigma_2]\delta K_2}{[(\sigma_2+\mu+d) + (1-\tau)](\theta\sigma_2 + \sigma_1K_2)\rho - K_1(1-\rho)\sigma_2}$$

$$\frac{\partial R_0}{\partial \mu_b} = \frac{\partial R_0}{\partial \mu_b} \times \frac{\mu_b}{R_0}$$

$$\frac{\partial R_0}{\partial \mu_b} = \frac{\Lambda v_1[\theta\sigma_2 + \sigma_1 K_2)\rho - K_1(1-\rho)\sigma_2]}{K_1 K_2(\mu+v)K} \times \frac{\mu_b K_1 K_2 K_3(\mu+v)K}{\Lambda v_1(\theta\sigma_2 + \sigma_1 K_2)\rho - K_1(1-\rho)\sigma_2} \quad (29)$$
$$= \mu_b^{\,2}$$

L. Numerical Solution of the Model with Homotopy Perturbation Method

Since we intend to numerically simulate the mathematical model, we intend to provide an approximate solution using the Homotopy perturbation method since there is no associated exact solution to the model. The analysis of the HPM will be given.

$$A(\alpha) = k(\tau), \ \tau \ni \lambda. \quad (29)$$

Subject to the boundary condition

$$\Omega(\alpha, \alpha_n) = 0 \ \tau \ni \Pi. \quad (30)$$

Operator $\lhd$ represents the differential operator, Ω denotes the boundary operator,

k(τ) is an analytic function, Ψ is defined domain bounded by Π, and α_n is a normal

vector derivative drawn externally from Ψ. Thus we can separate the operator $A(\alpha)$

into two:

$$\lhd(\alpha) = L_T(\alpha) + N_T(\alpha), \quad (31)$$

The operator $L_T(\alpha), N_T(\alpha)$ denote the linear and nonlinear term respectively such

that equation (34) implies:

$$L_T(\alpha) + N_T(\alpha) = k(\tau), \ \tau \ni \lambda.$$

We can construct a Homotopy for (34) so that

$$H(f,p) = (1-p)[L_T(f) - L_T(\omega_0)] + p[A(f) - k(r)] = 0$$

Where p is an embedding parameter which can undergo a deformation process of changing from [0,1]

Equation (41) is further simplified to obtain:

$$H(f,p) = L_T(f) - L_T(\alpha_0) + p[L_T(\alpha_0)] + p[N_T(\alpha_0)] + p[N_T(\alpha_0) - k(r)] = 0 \quad (32)$$

As $p \to 0$, equation (32) yields:

$$H(f,0) = L_T(f) - L_T(\alpha_0) = 0$$

And when $p \to 1$,

$$H(f,1) = A(f) - k(\tau) = 0.$$

We can naturally assume the solution (44) as a power series such that

$$f(t) = f_0(t) + pf_1(t) + p^2 f_2(t) + ...p^n f_n(t) \quad (33)$$

Evaluating (44) with (45), and comparing coefficients of equal powers of p,
The values of $f_0(t), f_1(t), f_2(t)$ are obtained by solving ordinary differential equations.
Thus, the approximate solution of (33) is:

$$f(t) = \lim_{p\to 1} f_n(t) = f_0(t) + f_1(t) + f_2(t) + ... \quad (34)$$

i. Saturation analysis and convergence

Here, to implement the application of the Homotopy perturbation method in solving the proposed model while taking care of the inhibitory parameter, unlike several other existing, we conduct an analysis of the saturation and further examine its convergence for usage.

Hence, consider $\frac{1}{1+\delta I}$, This can be expanded in series form such that

$$(1+\delta I)^{-1} = 1 - \delta I + \delta^2 I^2 - \delta^3 I^3 + \delta^4 I^4 - \delta^5 I^5 + 0(I^6) \quad (35)$$

Based on the convergence of this series it is obtained that,

$|\delta I| < 1$, that is $|\delta| < \frac{1}{|I|}$

To conduct numerical simulation on the mathematical model, we create the following correctional scheme for the model equation

$$(1-p)\frac{dS}{dt}+p\left(\frac{dS}{dt}-\Lambda-\omega R+(\mu+\lambda+v)S\right)=0$$

$$(1-p)\frac{dC}{dt}+p\left(\frac{dC}{dt}-\rho\lambda S+(\sigma_1+\theta+\mu+\phi)C\right)=0$$

$$(1-p)\frac{dI}{dt}+p\left(\frac{dI}{dt}-(1-p)\lambda S-\theta C+(\sigma_2+\mu+d)I+\frac{(1-\tau)rI}{1+\delta I}\right)=0 \quad (36)$$

$$(1-p)\frac{dR}{dt}+p\left(\frac{dR}{dt}-\frac{(1-\tau)rI}{1+\delta I}-\phi C+(\mu+\omega)R-vS\right)=0$$

$$(1-p)\frac{dB}{dt}+p\left(\frac{dB}{dt}-\sigma_1 C-\sigma_2 I+\mu_b B\right)=0$$

Simplifying the equation (39) yields:

$$\frac{dS}{dt}=p\big(\Lambda+\omega R-(\mu+\lambda+v)S\big)$$

$$\frac{dC}{dt}=p\big(\rho\lambda S-(\sigma_1+\theta+\mu+\phi)C\big)$$

$$\frac{dI}{dt}=p\left((1-p)\lambda S+\theta C-(\sigma_2+\mu+d)I-\frac{(1-\tau)rI}{1+\delta I}\right) \quad (37)$$

$$\frac{dR}{dt}=p\left(\frac{(1-\tau)rI}{1+\delta I}+\phi C-(\mu+\omega)R+vS\right)$$

$$\frac{dB}{dt}=p\left(\sigma_1 C+\sigma_2 I-\mu_b B\right)$$

The approximate solution of (37) can be assumed as:

$$S(t)=s_0(t)+ps_1(t)+p^2s_2(t)+p^3s_3(t)+\ldots p^ns_n(t)$$

$$C(t)=c_0(t)+pc_1(t)+p^2c_2(t)+p^3c_3(t)+\ldots p^nc_n(t)$$

$$I(t)=i_0(t)+pi_1(t)+p^2i_2(t)+p^3i_3(t)+\ldots p^ni_n(t) \quad (38)$$

$$R(t)=r_0(t)+pr_1(t)+p^2r_2(t)+p^3r_3(t)+\ldots p^nr_n(t)$$

$$B(t)=b_0(t)+pb_1(t)+p^2b_2(t)+p^3b_3(t)+\ldots p^nb_n(t)$$

Evaluating (53) using (54) and comparing coefficient of p^n

p^0 :

$$\dot{s}_0(t)=0,\quad \dot{c}_0(t)=0,\quad \dot{i}_0(t)=0,\quad \dot{r}_0(t)=0,\quad \dot{b}_0(t)=0$$

Solving (54) yields

$$s_0(t)=s_0,\ c_0(t)=c_0,\ i_0(t)=i_0,\ r_0(t)=r_0,\ b_0(t)=b_0$$

Similarly comparing the coefficients of p^1

$$\frac{ds_1}{dt}=\left(\Lambda+\omega r_1-(\mu+\lambda+v)s_1\right)$$

$$\frac{dc_1}{dt}=\left(\rho\lambda s_1-(\sigma_1+\theta+\mu+\phi)c_1\right)$$

$$\frac{di_1}{dt}=\left((1-p)\lambda s_1+\theta c_1-(\sigma_2+\mu+d)i_1-\frac{(1-\tau)ri_1}{1+\delta i_1}\right) \quad (39)$$

$$\frac{dr_1}{dt}=\left(\frac{(1-\tau)ri_1}{1+\delta i_1}+\phi c_1-(\mu+\omega)r_1+vs_1\right)$$

$$\frac{db_1}{dt}=\left(\sigma_1 c_1+\sigma_2 i_1-\mu_b b_1\right)$$

Solving the system (39) yields:

At p^1, the first iterations are obtained as

$$s_1(t)=(-\lambda s_0-\mu s_0+\omega r_0-vs_0+\Lambda)t$$

$$c_1(t)=(\lambda\rho s_0-\mu c_0-\phi c_0-\theta c_0-\theta c_0-c_0\sigma_1)t$$

$$i_1(t)=(\delta^2 r\tau i_0{}^3-\delta^2 ri_0{}^3-\delta\tau i_0{}^2+\delta ri_0{}^2-\lambda\rho s_0+r\tau i_0-di_0+\lambda s_0-\mu i_0-ri_0 \\ +\theta c_0-i_0\sigma_2)t$$

$$r_1(t)=(-\delta^2 r\tau i_0{}^3+\delta^2 ri_0{}^3+\delta r\tau i_0{}^2-\delta ri_0{}^2-r\tau i_0-\mu r_0-\omega r_0+\phi c_0+ri_0+vs_0)t \quad (40)$$

$$b_1(t)=(-b_0\mu_b+c_0\sigma_1+i_0\sigma_2)t$$

At p^2, the second iteration are obtained as

$$s_2(t) = -\frac{1}{2}t^2\begin{pmatrix} \delta^2\omega r\tau i_0^{\ 3} - \delta\omega r i_0^{\ 3} - \delta\omega r\tau i_0^{\ 2} + \delta\omega r i_0^{\ 2} + \omega r\tau i_0 - \lambda^2 s_0 - 2\lambda\mu s_0 + \lambda\omega r_0 \\ -2\lambda v s_0 - \mu^2 s_0 + 2\mu\omega r_0 - 2\mu v s_0 + \omega^2 r_0 - \omega\phi c_0 - \omega r i_0 + \omega v r_0 - \omega v s_0 \\ -v^2 s_0 + \Lambda\lambda + \Lambda\mu + \Lambda v \end{pmatrix}$$

$$c_2(t) = \frac{1}{2}t^2\begin{pmatrix} -\lambda^2\rho s_0 - 2\lambda\mu\rho s_0 + \lambda\omega\rho r_0 - \lambda\phi\rho s_0 - \lambda\rho\theta s_0 - \lambda\rho v s_0 - \lambda\rho s_0\sigma_1 + \Lambda\lambda\rho \\ +\mu^2 c_0 + 2\mu\phi c_0 + 2\mu\theta c_0 + 2\mu c_0\sigma_1 + \phi^2 c_0 + 2\phi\theta c_0 + 2\phi c_0\sigma_1 + \theta^2 c_0 + \\ 2\theta c_0\sigma_1 + c_0\sigma_1^{\ 2} \end{pmatrix}$$

$$i_2(t) = -\frac{1}{2}t^2\begin{pmatrix} 2\lambda\mu s_0 + \lambda v s_0 - 2r\tau i_0\delta\rho\lambda s_0 + 3r\tau i_0^{\ 2}\rho\lambda s_0 \\ +4r\tau i_0^{\ 3}\delta^2 d + 4r\tau i_0^{\ 3}\delta^2\mu + 4r\tau i_0^{\ 3}\delta^2\sigma_2 \end{pmatrix} + \frac{2}{3}\begin{pmatrix} -3r^2\delta^4 i_0^{\ 5} + 5r^2\delta^3 i_0^{\ 4} - 6r^2\delta^2 i_0^{\ 3} \\ +3r^2 i_0^{\ 2}\delta - r^2\tau^2 i_0 + 2r^2\tau i_0 + d\lambda s_0 \\ -2d\mu i_0 - 2dr i_0 - d\theta c_0 - 2d i_0\sigma_2 - 2\mu r i_0 \\ -+\lambda r s_0 + r\theta c_0 - 2r i_0\sigma_2 \end{pmatrix}$$

$$r_2(t) = \frac{1}{2}t^2\begin{pmatrix} (-\lambda v s_0 - 2\mu v s_0 - \omega\phi c_0 - \omega r i_0 - \omega v s_0 - 2r\tau i_0\delta\rho\lambda s_0 + 3r\tau i_0^{\ 2}\delta^2\rho\lambda s_0 - \delta^2\omega r i_0^{\ 3} \\ \delta\omega r i_0^{\ 2} + \omega r\tau i_0 + 3r\tau i_0^{\ 3}\delta^2 d + 4r\tau i_0^{\ 3}\delta^2\mu + 3r\tau i_0^{\ 3}\delta^2\sigma_2 + 3r\delta^2 i_0^{\ 2}\lambda s_0 + 3r\delta^2 i_0^{\ 2}\theta c_0 - 2r\tau i_0^{\ 2}\delta d \end{pmatrix} +$$
$$-\frac{1}{6}\begin{pmatrix} -3r^2\tau^2 i_0^{\ 5}\delta^4 + 6r^2\tau i_0^{\ 5}\delta^4 + 5r^2\tau^2 i_0^{\ 4}\delta^3 - 10r^2\tau i_0^{\ 4}\delta \\ -6r^2\tau^2 i_0^{\ 3}\delta^2 + 12r^2\tau i_0^{\ 3}\delta^2 - 3r^2\delta^4 i_0^{\ 5} - v^2 s_0 + \Lambda v - r^2 i_0 \end{pmatrix}$$

$$b_2(t) = \frac{1}{2}t^2(\mu_b^{\ 2} b_0 - \mu_b c_0\sigma_1 - \mu_b i_0\sigma_2 - r\tau\delta^2 i_0^{\ 3}\sigma_2 - r\delta^2 i_0^{\ 3}\sigma_2 - r\tau i_0^{\ 2}\delta\sigma_2 + r i_0^{\ 2}\delta\sigma_2 - \lambda\rho s_0\sigma_2 + r\tau i_0\sigma_2$$
$$d\tau i_0\sigma_2 - d i_0\sigma_2 + \lambda s_0\sigma_2 - \mu i_0\sigma_2 - r i_0\sigma_2 + \theta c_0\sigma_2 - i_0\sigma_2^{\ 2} + \lambda\rho s_0\sigma_1 - \mu c_0\sigma_1 - \phi c_0\sigma_1 - \theta c_0\sigma_1 - c_0\sigma_1^{\ 2}$$

III. Results and Discussion

From the simulation of iterative values of the model solution via homotopy perturbation method these are computed graphically with the help Maple-18 software as for respective compartments The results of Fig. 2 to Fig.6 represents the effects of the key parameters on the compartments of the model solution. Hence these are graphically illustrated below:

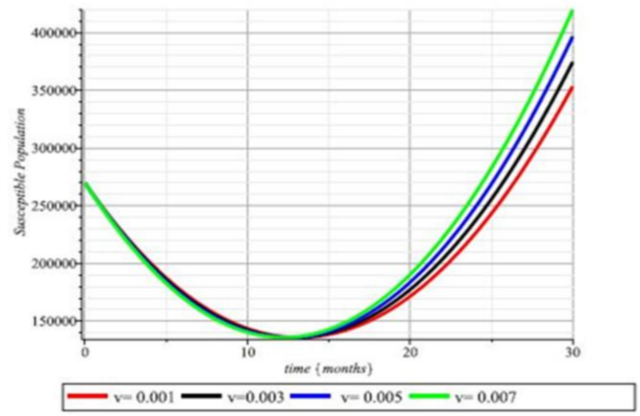

Fig 1: Analysis of Vaccination Uptake on Susceptible Population

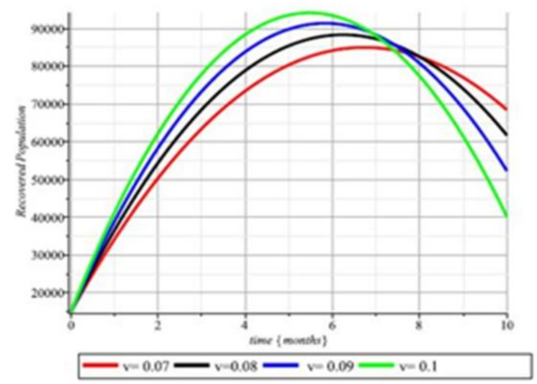

Fig 2: Dynamical Response of Vaccination Uptake on Recovered Population

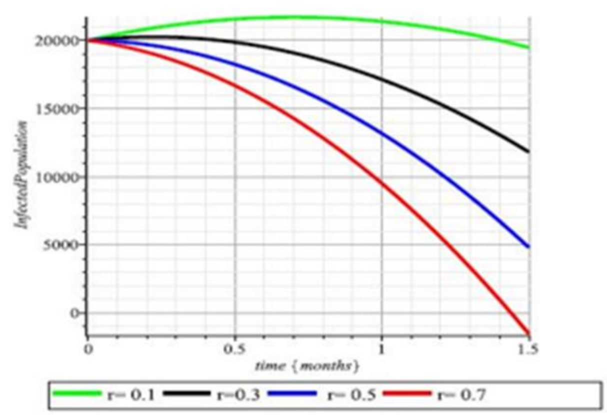

Fig 3: Dynamical Response of Infected Patients' Therapeutic Actions at 0.5 Rate of Drug Resistance

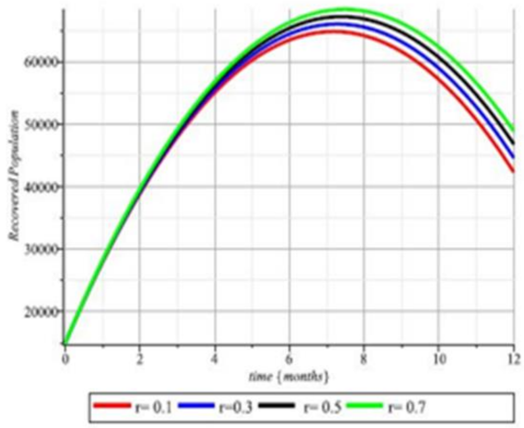

Fig 4: Dynamical Response of Recovered Patients' Therapeutic Actions at 0.5 Rate of Drug Resistance

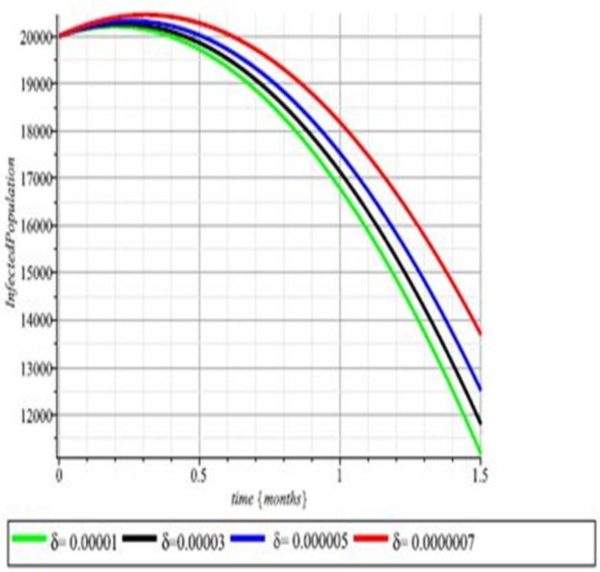

Fig 5: Dynamical Response of Infected Population to the Therapeutic at 0.5 Drug Resistant Rate

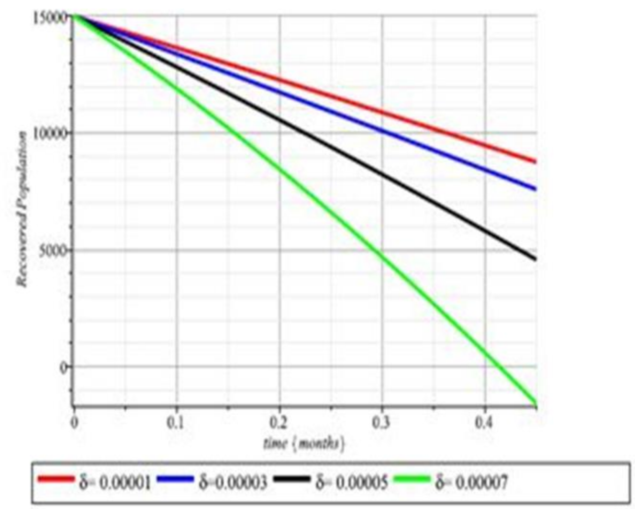

Fig 6: Dynamical response of recovered population to the therapeutic at 0.5 drug

A. Interpretation of the Graphs

Here, we discuss the outcomes of the study and the conducted numerical experiments. To begin, Table 4.11 Based on the results, we observed these parameters ϕ, σ_1, σ_2 *and* ρ have a positive sensitivity indices on R_0 as a consequence, increment in these values will raise R_0 and lowering these vales will decrease R_0. Thus, strategic ways applicable to lowering the rate of the parameters, specifically hose with higher index such as ϕ should be put in place. Furthermore, the index indicates that the discharge rate of bacteria from carriers and infections should be closely monitored to minimize the rate at which the environment get contaminated so as to avoid increase susceptibility of vulnerable population. Lastly, increasing the screening rate will lower the growth of R_0 and consequently reduce disease transmission.

Fig. 1 shows the dynamics of susceptible population such that as v increases from 0.0001 to 0.0007, the susceptible population increases. The implication of this is that increase in vaccination will increase the population of the susceptible. We conclude that the number of individuals who have been with typhoid fever disease before the application of vaccine has gone down due to impact of vaccine on infected individuals.

Fig. 2 reveals the simulated results the model variables at 50% using antibiotic resistance on Transmission dynamics of Typhoid fever, and as v increases from 0.07 to 0.1. it clearly show that recovered population has increased to maximum level . Therefore we conclude that applying vaccine in eradicating typhoid fever disease from the community is effective in a specified period of time.

Fig. 3 clearly shows that the infectious population drastically reduces to bare minimum as therapy implementation increases. Consequently this affirm that this strategy is effective in eradicating the disease from the community as time progresses.

Fig. 4 reveals that despite the level of antibiotic drug resistance of infected individuals a great response is observed concerning the efficacy of treatment as infected population drastically reduces and Recovered individuals grows to maximum level.

Fig. 5 It clearly shows that the infected population has reduced to bare minimum at the end of the implementation of saturated treatment response. Therefore we conclude that this strategy will eradicate typhoid fever disease from the community as time progresses.

Fig. 6 clearly shows that the recovered population has reduced to bare minimum as saturated treatment response increases from 0.00001 to 0.00007, which indicate that saturated treatment response has reach it saturation point and at that point it can aid complication in treatment.

IV. Conclusion

This research presents a comprehensive mathematical model for understanding typhoid fever transmission, considering treatment response, vaccination and antibiotic resistance. Through qualitative analysis, we establish the model's epidemiological reliability, demonstrating stability conditions for disease control. Sensitivity analysis identifies key parameters influencing disease spread, while numerical simulations underscore the role of high vaccination coverage in achieving herd immunity. The results highlight the necessity of integrated strategies, combining vaccination, effective treatment and responsible antibiotic use, to curb typhoid fever. Ultimately, this research emphasizes the importance of addressing treatment saturation, vaccine efficacy and antibiotic resistance for sustainable disease management.

We recommend a multifaceted approach to typhoid fever control, emphasizing widespread vaccination, optimized treatment strategies and responsible antibiotic use. Health care practitioners should prioritize vaccine accessibility also must ensure effective treatment and monitor antibiotic resistance. Public health initiatives should focus on awareness campaigns to promote hygiene and vaccination. Further research should refine mathematical models to enhance disease prediction and intervention effectiveness.

Financial Support

No financial support is obtained in the course of this research

Acknowledgement

Authors acknowledged the tremendous efforts of all anonymous reviewers that will improve this manuscript.

Declaration

The authors of the paper declared that there are no conflicts of interest.

References

[1] A. I. Alaje, M. O. Olayiwola, M. O. Ogunniran, J. A. Adedeji and K. A. Adedokun "Approximate Analytical Methods for the Solution of Fractional Order Integro-differential Equations". *Nigerian Journal of Mathematics and Applications* Volume 31, Number 2, 2021, pp 175 – 190

[2] Adebimpe O., Waheed A. A., Gbadamosi B." Modeling and Analysis of an SEIRS Epidemic Model With Saturated Incidence". *Int. Journal of Engineering Research and Application*. Volume 18, number 7, 2013, pp 15 – 26

[3] Adebisi A. F, Peter O. J , Ayoola T. A, Oguntolu F. A, Ishola C. "Approximate Solution of Typhoid Fever Model by Variational Iteration Method". *Journal of Science, Technology & Education (JOSTE)*; Volume 6, Number 3, 2018, pp 164 –177

[4] Adedapo Ismaila Alaje, Morufu Oyedunsi Olayiwola, Kamilu Adewale Adedokun, Joseph Adeleke Adedeji and Asimiyu Olamilekan Oladapo. "Modified Homotopy perturbation method and its application to analytical solutions of fractional-order Korteweg–de Vries equation". *Beni-Suef Univ J Basic Appl Sci* Volume 34, Number 1, 2022, pp 9 –20

[5] Akeem O. Yunus, Morufu O. Olayiwola, Kamilu A. Adedokun, Joseph. A. Adedeji and Ismaila A. Alaje "Mathematical analysis of fractional-order Caputo's derivative of coronavirus disease model via Laplace Adomian decomposition method, Yunus *et, al. Beni-SuefUniv J Basic Appl Sci,* Volume 17, Number 4, 2022, pp 18 –33

[6] Getachew Teshome Tilahun, Oluwole Daniel Makinde, And David Malonza "Modelling and Optimal Control of Typhoid fever disease with cost effective strategies".*Hindawi computational and Mathematical methods in medicine*, Volume 2, Number 5, 2023, pp 119 –134

[7] Hamadjam Abboubakar & Reinhard Racke "Mathematical modelling and optimal control of typhoid fever" *Konstanzer Online-Publikations-System (KOPS).* Volume 8, Number 3, 2024, pp 23–41

[8] Kolawole M. K., Olayiwola M. O "On the Numerical Effect of Saturation Terms on the Susceptible Individual in Susceptible-Exposed-Infected-Recovered- Susceptible (SEIRS) Epidemic Model". *Computing, information Systems, Development Informatics & Allied journal* Volume 7, Number 2, 2023, pp 83- 90.

[9] Kolawole, M. K., Ayoola T.A., Abdulrazaq R."Modelling Typhoid Fever with General Knowledge,Vaccination and Treatment for SusceptibleIndividual", *International Journal of Research and Innovation in Applied Science (IJRIAS)*. Volume 9, Number 11, 2022, pp 73- 91

[10] Kolawole M. K., Olayiwola M. O "Simulation of the effect of disease transmission coefficient on the susceptible-exposed-infected-recovered (SEIR))epidemic model using variational iteration method". *Anale. Serial Informatică*. Volume 18, Number 6, 2024, pp 3- 24

[11] Tawakalt Abosede Ayoola, Mutairu Kayode Kolawole, and Amos Oladele Popoola. "Effects of Acceptance of Enlightenment on COVID-19 Transmission using Homotopy Perturbation Method". *Jambura Journal of Biomathematics*, Volume 3, Number 2, 2022, pp 39–48

[12] Tawakalt Abosede Ayoola, Mutairu Kayode Kolawole, and Amos Oladele Popoola. Mathematical Model of Covid-19 Transmission Dynamics with Double Dose Vaccination".. *Tanzania Journal of Science*. Volume 48, Number 23, 2022, pp 48–56.

[13] Zhang, Juan and Ma, Zhien "Global dynamics of an SEIR epidemic model with saturating contact rate", *Math Biosci.* Volume 21, Number 16, 2018, pp 15-32.

[14] Zhao, Xiaoqiang "Dynamical systems in population Biology". *CMS books in Mathematics*, Volume 26, Number 3, 2021, pp 18–29

[15] Tawakalt Abosede Ayoola, Mutairu Kayode Kolawole, and Amos Oladele Popoola. "Mathematical Model of Covid-19 Transmission Dynamics with Double Dose Vaccination". *Tanzania Journal of Science*. Volume 29, Number 9, 2022, pp 38–49.

[16] Ahmed Ademola, Bozkurt F, Ali Y, Abdeljawad T, Adem K, Al Mdallal Q "A Fractional Order Model of COVID-19 Considering the Fear Effect of the Media and Social Networks on the Community". *Chaos Solitons Fractals* Volume 1, Number 5, 2021, pp 52:78

[17] Virginia E. Pitzer, James Meiring, Frederick P. Martineau, Conall H. Watson, Gagandeep Kang, Buddha Basnyat, and Stephen Bake. "The Invisible Burden: Diagnosing and Combatting Typhoid Fever in Asia and Africa".*Beni-Suef University Journal of Basic and Applied Sciences*, Volume 11, Number 1, 2022, pp 98-118.